JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF   SCIENCE

ACTUARIAL

3RD   YEAR 1ST SEMESTER 2022/2023

REGULAR (MAIN)

COURSE CODE: WAB 2306

COURSE TITLE: **FINANCIAL ECONOMICS 1**

EXAM VENUE: **STREAM: (BSc Actuarial Science)**

DATE: **9/12/2022**   **EXAM SESSION: 9.00-11.00AM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION 1

a. Define the following terms

    i.   Portfolio **(2MARKS)**

    ii.   Non satiation **(2MARKS)**

    iii.   Risk averse investor **(2MARKS)**

    iv.   Utility **(2MARKS)**

b. Each year, Mr. Wills is offered the opportunity to invest KSH10, 000 in a risk fund. If successful, at the end of the year he will be given back KSH20, 000. If unsuccessful, he will be given back only KSH5, 000. There is a 50% chance of either outcome. Calculate the expected rate of return per annum on the investment. **(2MARKS)**

c. List five assumptions underlying mean-variance portfolio theory. **(5MARKS)**

d. Investor A has an initial wealth of $100, which is currently invested in a non-interest-bearing account, and a utility function of the form:

$$U(w) = \log(w)$$

Where w is the investor's wealth at any time.

Investment Z offers a return of -22% or $+28\%$ with equal probability.

    I.   What is Investor A's expected utility if nothing is invested in Investment Z?

    (**2MARKS**)

    II.   What is Investor A's expected utility if they're entirely invested in Investment Z?

    (2MARKS)

e. What proportion α of wealth should be invested in Investment Z to maximize expected utility? What is Investor A's expected utility if they invest this proportion in Investment Z? **(2MARKS)**

f. An insurance company will be required to make a payout of £500 on a particular risk event, which is likely to occur with a probability of 0.4. The utility for any level of wealth, w, is given by:

$$U(w) = 4{,}000 + 0.5w$$

The insurer's initial level of wealth is £6000. Calculate the minimum premium the insurer will require in order to take on the risk.

(5MARKS)

g. Define first-order and second-order stochastic dominance. **(4MARKS)**

QUESTION 2

a. Investment returns (% *pa*), X, on a particular asset are modelled using a probability distribution with density function:

$$f(x) = 0.00085(110 - (x - 10)^2) \text{ where } -5 \leq x \leq 15$$

Calculate the

i. mean return **(4MARKS)**

ii. Variance of return. **(4MARKS)**

iii. downside semi-variance of return **(4MARKS)**

iv. Value at Risk (VaR) over one year with a 95% confidence limit for a portfolio consisting of KSH10, 000,000 invested in the asset. **(4MARKS)**

v. shortfall probability where the benchmark return is 0% *pa* **(4MARKS)**

QUESTION 3

a. Define the following measures of investment risk:

I. variance of return **(2MARKS)**

II. downside semi-variance of return **(2MARKS)**

III. shortfall probability **(2MARKS)**

IV. Value at Risk. **(2MARKS)**

b. Investment returns (% *pa*), *X* , on a particular asset are modelled using the probability distribution:

X	Probability
−7	0.04
5.0	0.90
8.9	0.06

Calculate the mean return and variance of return. **(4MARKS)**

c. The returns from an investment in Jubilee holding insurance are assumed to conform to the fixed rate model with the distribution of rates as specified below:

    0.06   With probability 0.2

    0.08   With probability 0.7

    0.10   With probability 0.1

  I.   Calculate the expected accumulated value at the end of 5 years of an initial investment of KSH75, 000. **(2MARKS)**

  II. Calculate the accumulated value at the mean rate of return. **(3MARKS)**

  III. Calculate the variance of the accumulated value of the investment **(3MARKS)**

QUESTION 3

a. List three capital investment appraisal methods used to test viability of a financial project. **[3 marks]**

b. An investor has the utility function $U(w) = - exp\left(-\frac{w}{100}\right)$ Determine whether the investor exhibits increasing, constant or decreasing absolute and relative risk aversion. **[5 marks]**

c. £200 is invested for 12 years. In any year the yield on the investment will be 3% with probability 0.25, 5% with probability 0.6 and 6% with probability 0.15, and is independent of the yield in any other year.

  I.   Calculate the mean accumulation at the end of 12 years. **(3MARKS)**

II.   Calculate the standard deviation of the accumulation at the end of 12 years.

(3MARKS)

d. A stochastic model of investment returns assumes that the annual rates of return in different Years are independent and identically distributed normal random variables with mean 8% and Standard deviation 2%. Calculate the mean and standard deviation of the accumulated value, at time 2, of an initial Investment of £10,000.

(6MARKS)

QUESTION 4

a. An insurer with initial wealth of KSH2, 000,000 and a utility of $U(x) =\log(x)$ is designing a policy to cover damages of KSH500, 000 that occur with probability 0.5.Calculate the minimum premium that the insurer can charge for the policy.   **(6MARKS)**

b. Suppose Investor A has a *power* utility function with $\gamma = 0.9$, whilst Investor B has a power utility function with $\gamma = 0.75$.
   i.   Which investor is more risk-averse ?   **(3MARKS)**
   ii.   Suppose that Investor B has an initial wealth of 100 and is offered the opportunity to buy Investment X for 100, which offers an equal chance of a payout of 110 or 92. Will the Investor B choose to buy Investment X?

(3MARKS)

c. Consider the two risky assets, A and B, with cumulative probability distribution functions:

$$F_A(w) = w$$

$$F_B(w) = w^{\frac{1}{2}}$$

In both cases, $0 \leq w \leq 1$.

i.   Show that A is preferred to B on the basis of first-order stochastic dominance.**(4MARKS)**

ii. Verify explicitly that A also dominates B on the basis of second-order stochastic dominance.   **(4MARKS)**

QUESTION 5

a. State five assumptions of black Scholes model.   **(5MARKS)**

b. A building society issues a one-year bond that entitles the holder to the return on a weighted-average share index (ABC kenya limited) up to a maximum level of 30% growth over the year. The bond has a guaranteed minimum level of return so that investors will receive at least x% of their initial investment back. Investors cannot

redeem their bonds prior to the end of the year. The volatility of the ABC Kenya limited index is 30% *pa* and the continuously compounded risk-free rate of return is 4% *pa*. Assuming no dividends, use the Black-Scholes pricing formulae to determine the value of *x* (to the nearest 1%) that the building society should choose to make neither a profit nor a loss. **(15MARKS)**