JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

3RD YEAR 1ST SEMESTER 2022/2023 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE:WAB 2309

COURSE TITLE:THEORY OF ESTIMATION

EXAM VENUE: LAB 17 **STREAM: (BSc. Actuarial)**

DATE: 15/12/2022 EXAM SESSION: 15.00-17.00PM

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (20 MARKS)

a) Explain clearly the following terms as used in theory of estimation.

   i. Sufficiency
   ii. Weak Consistency
   iii. Completeness
   iv. Uniformly Minimum Variance Unbiased Estimator.
   v. Most Efficient Estimator
   vi. Unbiasedness **[6 Marks]**

b) Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample of size n from the Poisson population with parameter λ. Show that $T_n = \frac{\bar{x}}{1+\frac{1}{n}}$ is consistent for λ. **[6 Marks]**

c) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from the exponential distribution with pdf $f(x;\theta) = \frac{1}{\theta} e^{-x/\theta}$. Show that $\bar{X}$ is unbiased for θ. **[6 Marks]**

d) Given two estimators of population mean (μ) as $T_1 = \frac{(X_1+2X_2+X_3)}{4}$ and $T_2 = \frac{(X_1+X_2+X_3)}{3}$ where X_1, X_2 and X_3 are from $N(\mu, \sigma^2)$distribution. Prove that $T_2$is more efficient than T_1 . **[6 Marks]**

e) Let $X_1, X_2, \ldots, X_n$ be iid $Uniform\ [a, b]$ random variables with a known. Find an unbiased estimator for b. **[6 Marks]**

QUESTION TWO (20 MARKS)

a) Let $X_1, X_2, \ldots, X_n$ be iid random variables from the $N(\mu, \sigma^2)$ distribution. Find the maximum likelihood estimators of μ and σ^2 **[11 Marks]**

b) Let $X_1, X_2, \ldots, X_n$ be iid random variables from a uniform distribution on the interval $(\theta - 1, \theta + 1)$.

   i. Find the method of moments estimator of θ.
   ii. Is the obtained estimator unbiased for θ ? **[9 Marks]**

QUESTION THREE (20 MARKS)

a) Use the Lehmann Scheffe method of construction of minimal sufficient statistics to find the minimal sufficient statistic for $\theta = (\mu, \sigma^2)$ given $X_1, X_2, \ldots, X_n$ are iid random variables from (μ, σ^2)**.** **[9 Marks]**

b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(\mu, \sigma^2)$. Find Fisher's information $I(\mu)$necessary for estimation of the Population mean(μ) hence the associated Cramer-Rao lower bound. **[11.Marks]**

QUESTION FOUR (20 MARKS)

Let $X_1, X_2, \ldots, X_n$ be a random sample from a population having probability density function;

$$f(x) = \begin{cases} \frac{2}{\theta^2}(\theta - x), 0 < x < \theta \\ 0, \quad Otherwise \end{cases}$$

Let $T = \frac{3}{2}\bar{x}$ be an estimator of θ.Find the Root Mean Squared Error of the estimator T

[20 Marks]

QUESTION FIVE (20 MARKS)

a) Let $X_1, X_2, \dots, X_n$ be iid Poisson random variables with $f(x,\theta) = \frac{e^{-\theta}\theta^x}{x!} x = 0,12, \dots$
Show that the maximum likelihood estimator for $\theta$is $\bar{X}$. **[8 Marks]**

b) Let $X_1, X_2, \dots, X_n$ be a random sample of n observations from a population having p.d.f

$$f(x) = \begin{cases} \frac{2x}{\theta^2}, 0 \le x \le \theta \\ 0, \quad otherwise \end{cases}$$

Check if $T = \frac{3\bar{x}}{2}$ is consistent for θ **[12 Marks]**