JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE

1ST YEAR 1ST SEMESTER 2022/2023ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WMB 9107

COURSE TITLE: BASIC MATHEMATICS

EXAM VENUE: **STREAM: EDUCATION, ACTUARIAL**

DATE: 19/12/2022 **EXAM SESSION: 9.00-11.00AM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

<u>QUESTION ONE</u> (30 marks)

a) Prove the identity
$\frac{1+\tan\theta}{1+cot\theta} = \tan\theta$(5marks)

b) i) Given that $A = \{1,0\}$ and $B = \{x, y, z,\}$, show that the $A \times B \neq B \times A$ (2marks)
ii) Use the quadratic formula to find the real solutions, if any, of the equation
$3x^2 + 2 = 4x$ (3marks)

c) Solve the equation: $3^{2x} + 3^{x+1} - 4 = 0$(5marks)

d) i) Use Binomial theory to determine the expansion of $\left(1 - \frac{x}{4}\right)^3$ (3marks)

ii) In how many different ways can the letters of the word 'MORAA' be arranged so that the vowels always come together?(2marks)

e) Otieno starts a part-time job on a salary of $\$9000$ per year, and this increases by an annual increment of $\$1000$. Assuming that, apart from the increment, Otieno's salary does not increase, find
i) his salary in the 12^{th}year (2marks)
ii) the length of time he has been working when his total earnings are $\$100\ 000$ (3marks)

f) Write $(1 + i)^5$ in the standard form $a + bi$ (5marks)

<u>QUESTION TWO</u> (20 marks)

a) Let f, g and h be the functions from the set of integers to the set of integers defined by $f(x) = 2x, g(x) = x^2$ and $h(x) = \frac{1}{x}$. Find:
i)$(fog)(x)$ (2marks)

ii)$(goh)(x)$ (2marks)

iii)$(fogoh)(x)$(2marks)

b) Let $f(x) = x^2 + 2, \forall x \in \mathbb{R}, x \neq 0$. Find:
i) $f^{-1}(x)$ (3marks)
ii) $f(5)$ and $f^{-1}f(5)$ (3marks)

c) Solve the equation $1 + sin\theta = 2cos^2\theta$ for θon the interval$0^\circ \leq \theta \leq 360^\circ$ (5marks)

d) Find the remainder when $f(x) = 4x^3 + 2x^2 + 3x + 1$ is divided by $(2x - 1)$ (3marks)

<u>QUESTION THREE</u> (20 marks)

a) Find the power set of $D = \{x: x \in \mathbb{N}\ and\ x^2 - 4x + 4 = 0\}$ (4marks)

b) Let $U = \{1, \dots, 9\}$ be the universal set and
$A = \{x: x \text{ is a prime number less than } 10\}$

$B = \{x: x \text{ is an even number less than } 10\}$and $C = \{2 < x < 7\}$. Find;

i)$(A \cup C)^c$ (2marks)

ii)$A^c \cap (B \backslash C)$ (3marks)

iii) $(A \Delta B)^c$(3marks)

c) Draw the Venn diagram and shade the region corresponding to
$(X \cap Y) \cap Z^c$ (2marks)

d) Given that the universal set $U = \{a, b, c, d, e, f, g, h, i, j\}$, $P = \{a, b, c\}$,
$Q = \{d, h, i, j\}$

Using the above sets, show that $(P \cap Q)^c = P^c \cup Q^c$ (6marks)

<u>QUESTION FOUR</u> (20 marks)

a) In a group of 60 students, 25 play table tennis, 16 do swimming and 22 play cricket, 8 play table tennis and do swimming, 6 play cricket and do swimming, 5 play table tennis and cricket, and 12 students do not play any of these games.

i) Draw a Venn diagram to represent the above information. (4marks)

ii) how many play table tennis, do swimming and play cricket?(2marks)

iii) how many play table tennis but not cricket?(2marks)

iv) how many play table tennis and cricket but not do swimming?(2marks)

b) Use Cramer's rule to solve the following systems of linear equations

$$2x + y + z = 1$$
$$3x + z = 4$$
$$x - y - z = 2$$

(10marks)

QUESTION FIVE (20 marks)

a) Kai is training for a marathon. Her training includes a run every Saturday starting with a run of 5 km on the first Saturday. Each Saturday she increases the length of her run from the previous Saturday by 2 km.

   i) Show that on the 4th Saturday of training she runs 11 km. (1mark)

   ii) Find an expression, in terms of n, for the length of her training run on the nth Saturday. (2marks)

   iii) Show that the total distance she runs on Saturdays in n weeks of training is $n(n + 4)$ km. (3marks)

   On the $n$thSaturday Kai runs 43 km.

   iv) Find the value of n. (2marks)

   v) Find the total distance, in km, Kai runs on Saturdays in n weeks of training. (2marks)

b) The third term of a geometric sequence is 324 and the sixth term is 96.

   (i) Show that the common ratiois $\frac{2}{3}$. (3 marks)

   (ii) Find the first term of the sequence. (2 marks)

   (iii) Find the sum of the first 15 terms. (3 marks)

   (iv) Find the sum to infinity of the sequence (2 marks)