JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE

3TH YEAR 1ST SEMESTER 2022/2023 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: WMB 9301

COURSE TITLE: REAL ANALYSIS

EXAM VENUE: **STREAM: EDUCATION, ACTUARIAL**

DATE: 16/12/2022 **EXAM SESSION: 15.00-17.00PM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 marks COMPULSORY)

a) Write the general formula for the following sequences (2 marks)

i) $^1/_2, ^1/_4, ^1/_8, ^1/_{16}, \dots$

ii) $-^3/_5,\ 5,\ ^7/_6, ^9/_{13}, ^{11}/_{22}, ^{13}/_{33}, \dots$

b) Prove that a sequence has at most one limit. (5 marks)

c) Prove that every convergent sequence of real numbers is bounded. (5 marks)

d) Describe the Comparison test theorem for a series and giving an example . (4 marks)

e) Prove that the sequence $\{\frac{n^2+1}{n^2+n}\}$ converges to 1. (4 marks)

f) Let $D = [0, 1] and\ f_n(x) = x^n$. Illustrate using a graph that the function f(x) converges pointwise but not uniformly for the function $f(x) = \begin{cases} 0\ if\ o \leq x \leq 1 \\ 1\ if\ x = 1 \end{cases}$ (4 marks)

g) Let $A = \{x \in \mathbb{Z}: -2 < x < 5\}$ $B = \{-1, 0, 1, 8\}$. Find $|A \triangle B|$ and card(A).(3 marks)

h) Prove that the $\lim_{n\to\infty} \frac{5n+5}{n} = 5$. (3 marks)

QUESTION TWO(20 marks)

a) Show that $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ (4 marks)

b) Define a cluster point in a metric space. (2 marks)

c) Show that a uniformly convergent sequence is convergent. (5 marks)

d) Prove that the set of real numbers in the interval (0, 1) is not countable. (9 marks)

QUESTION THREE(20 marks)

a) Show that the set $C[0, 1]$ of real-valued continuous functions defined on [0, 1] is a metric space with respect to the metric defined as $d(f, g) = \max\{|f(x) - g(x)|: x \in [0, 1]\}$where $f, g \in C[0, 1]$. (10 marks)

b) Determine whether the sequence $\frac{1}{3^n}$ is Cauchy. (7 marks)

c) Define $f, g: [0, 1] \to \mathbb{R}byf(x) = 2x$, $g(x) = 2x + 1$. Find (3 marks)

i) Sup f

ii) Inf f

iii) $f \circ g$

QUESTION FOUR(20 marks)

a) Compute the distance $d_1(f, g)$ and $d_\infty(f, g)$ when $f, g \in C[0, 1]$ are functions defined by $f(x) = x\ and\ g(x) = x^3$. (6 marks)

b) Let (X, d) be a metric space. Given a point $x \in X$ and a real number $r > 0$. Show that $A = \{y \in X: d(x, y) < r\}$ is an open set in X. (4 marks)

c) Prove that a bounded function $f: [a, b] \to \mathbb{R}$ is integrable if and only if for each $\varepsilon > 0$ there exist a partition P such that $U(f, P) - L(f, P) < \varepsilon$. (10 marks)

QUESTION FIVE(20 marks)

a) Determine the Riemann integral for the function$f(x) = x$ on$[0, 1]$ over the partitions $P = \{x_i = \frac{i}{4}, i = 0, 1, \dots, 4\}andQ = \{x_j = \frac{j}{8}, j = 0, 1, \dots, 8\}$. (4 marks)

b) Show that the series $\sum_{n=1}^{\infty}(\frac{1}{2})^n$ converges. (3 marks)

c) State 3 properties of the integral of a step function (3 marks)

d) Suppose that $\lim_{n\to\infty} x_n = land\lim_{n\to\infty} y_n = h$. Show that the $\lim_{n\to\infty}(x_n + y_n) = l + h$. (6 marks)

e) State the following terms as used in real analyssis (4marks)

i) Squeeze lemma

ii) Bolzano-Weirstrass theorem