JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION SCIENCE /ARTS

4TH YEAR 1ST SEMESTER 2022/2023 ACADEMIC YEAR

(MAIN)

COURSE CODE: WMB 9405

COURSE TITLE: TOPOLOGY

EXAM VENUE: **AH/LAB1/LAB 2** **STREAM: (BSc. EDS/ARTS)**

DATE: **9/12/2022** **EXAM SESSION: 9.00-11.00AM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 marks)Complulsory

a) Define the following terms:

i) A metric space as used in topology. (3mks)

ii) Neighbourhood (2mks)

b) Determine the neighbourhood$N(6,2)$ defined on

i) The usual metric space. (2mks)

ii) thediscrete metric space. (3mks)

c) Let $X = \{1,2,3\}$ and $\tau = \{\emptyset, \{1\}, \{2\}, \{1,3\}, \{2,3\}, X\}$.Determine whether τ is a topology on X or not. (5mks)

d) Let $X = \{1,2,3\}$ and $\tau = \{\emptyset, \{1\}, \{3\}, \{1,3\}, \{2,3\}, X\}$.If $A = \{1,2\}$. Find

i) $\text{Int}(A)$. (4mks)

ii) Limit points of A. (3mks)

e) Given that $X = \{1,2,3,4,5\}$ and $\tau = \{\emptyset, \{1\}, \{3\}, \{1,3\}, \{3,4\}, \{1,3,4\}, X\}$. If $A = \{1,2,4\}$, find the subspace topology τ_A on A. (3mks)

f) Let (X, d) be a metric space. Show that the whole space X is both open and closed. (5mks)

QUESTION TWO (20 marks)

a) Let $X = \mathbb{R}$. Define a metric $\rho: \mathbb{R} \times \mathbb{R} \dashrightarrow \mathbb{R}$by $\rho(x, y) = |x - y|, \forall\, x, y \in \mathbb{R}$. Show that ρ is indeed a metric on $\mathbb{R}$. (10mks)

b) In $(\mathbb{R}, d)$, where $d$is the usual metric on $\mathbb{R}$, a subset $A$of $\mathbb{R}$ is given by $A = (1,3] \cup (5,7) \cup \{9\}$. Find

i) Ainterior. (4mks)

ii) Aexterior. (4mks)

iii) The boundary of A. (2mks)

QUESTION THREE (20 marks)

a) If (X, d) is a metric space and $A = \{A_\alpha : \alpha \in I)$ is arbitrary family of open sets. Then show that $\cup A_\alpha : \alpha \in I$ is open in X. (5mks)

b) Show that finite intersection of open sets is open (5mks)

c) Show that arbitrary intersection of open sets need not be open (5mks)

d) Consider *B*={a,b,c,d}. Is $\mathcal{B}$ ={{a,b},{b,c,d}} a base for any topology on *B*? Explain. (5mks)

QUESTION FOUR (20 marks)

a) Let X be a non empty set and Let $A \subseteq X$. Let $\tau = \{U: A \subseteq U\}$.Determine whether τ is a topology on X or not. (8mks)

b) Given that $Y = \{x, y, z\}$, $\tau_1 = \{\emptyset, \{x\}, \{z\}, \{x, z\}, \{y, z), Y\}$ and $\tau_2 = \{\emptyset, \{x\}, \{x, y\}, \{x, z), Y\}$. Determine whether$\tau_1 \cup \tau_2$ and $\tau_1 \cap \tau_2$ are topologies on Y or not. (12mks)

QUESTION FIVE (20 marks)

a) Define the term continuity as used in topological spaces.

b) Let $X = \{1,2,3.4\}$, $Y = \{5,6,7,8\}$, $\tau_X = \{\emptyset, \{1\}, \{2\}, \{1,2\}, \{1,2,4), X\}$and $\tau_Y = \{\emptyset, \{5\}, \{7\}, \{5,7\}, \{5,6,8), Y\}$. Let $f: X \to Y$ be defined as $f(1) = 5, f(2) = 7, f(3) = 8$ and $f(4) = 6$. Determine whether f is continuous or not.(8mks)

c) Let (X, d) be a metric space. Show that finite union of closed sets in X is also closed. (6mks)

d) Describe the τ_2-axiom and hence define a Haussdorff space. (3mks)

e) Show that the discretetopological space is a τ_2- space. (3mks)