JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

2022/2023 EXAMINATIONS

MAIN SPECIAL

COURSE CODE: SPB 9220
COURSE TITLE: MATHEMATICAL METHODS FOR PHYSICS II
EXAM VENUE: **STREAM: EDUCATION**
DATE: **EXAM SESSION:**
TIME: 2:00 HRS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

SECTION I (30 MARKS)

Question 1 (Compulsory)

(a) (i) State the fundamental theorem of algebra. (1 mark)

(ii) Express in polar form and draw in an Argand diagram, the conjugate of z = 3 + 4i. (3 marks)

(b) Work out the Laurent series expansion for the function

$$f(z) = \frac{1}{z-1}$$

in the region $|z| > 1$. (2 marks)

(c) Evaluate f_x and f_y for the function $f(x, y) = 3x^2y^3 + 2xy^2 - xy + 2x - 3$ at the point (2, −1). (3 marks)

(d) (i) Define the pole of a function. (1 mark)

(ii) State the singularities in the complex function

$$f(z) = \frac{1}{(z-2)(z-3)}. \quad \text{(2 marks)}$$

(e) Use Cramer's rule to solve the simultaneous equations

$$\begin{aligned} 2x_1 + 4x_2 + 3x_3 &= 4 \\ x_1 - 2x_2 - 2x_3 &= 0 \\ -3x_1 + 3x_2 + 2x_3 &= -7. \end{aligned}$$

(3 marks)

(f) Evaluate $\left(-\sqrt{3} - i\right)^{-6}$ (3 marks)

(g) Find the determinant of $B = \begin{pmatrix} [illegible] \\ 0\ -2 \\ 4\ -3 \\ 1\ -2 \end{pmatrix}$.

(h) Find the general solution of the differential equation $\frac{dy}{dx} + by = 0.$ (3 marks)

(i) Show that in a Fourier series of a function f(x) in the interval [-π, π], the Fourier coefficient a_0 is given by

$$a_0 = \frac{1}{2\pi} \int f(x)dx \quad \text{(4 marks)}$$

Question 2 (20 marks)

(a) If $u = e^{x-y-2z}$,show that:

(i) $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ (4 marks)

(ii) $\frac{\partial^2 u}{\partial y \partial z} = \frac{\partial^2 u}{\partial z \partial y}$ (4 marks)

(b) Show that the function f(z) = $e^{-y} \cos x + ie^{-y} \sin x$ satisfies the Cauchy-Riemann conditions, hence find $f'(z)$. (8 marks)

(c) Find the total differential of each of the following functions:

(i) $z = x^2 - 3xy + y^2 + 2y$ (2 marks)

(ii) $z = cos^2(x - y)$ (2 marks)

Question 3 (20 marks)

(a) Determine a matrix P that diagonalizes $A = \begin{pmatrix} 0 & -2 \\ \text{[illegible]} \\ \text{[illegible]} \end{pmatrix}$. (13 marks)

(b) Investigate whether the following linear system of equations can be solved using Cramer's rule.

$$2x_1 + x_2 + x_3 = 6$$
$$3x_1 + 2x_2 - 2x_3 = -2$$
$$x_1 + x_2 + 2x_3 = 4$$

(7 marks)

Question 4 (20 marks)

(a) Find the linearization of $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$ at the point (3, 2). (9 marks)

(b) Show that the set $S = \left\{[0,1,0], \frac{1}{\sqrt{2}}[1,0,1] \frac{1}{\sqrt{2}}[1,0,-1]\right\}$ is orthonormal. (4 marks)

(c) Find the Laurent series expansion of

$$f(z) = \frac{1}{(z-1)(z-2)}$$

in the region $|z| < 1$. (7 marks)

Question 5 (20 marks)

(a) Given the basis $S = \{\vec{X_1}, \vec{X_2}, \vec{X_3}\}$ for $\mathbb{R}^3$ where

$$\overrightarrow{X_1} = [1,1,1], \overrightarrow{X_2} = [-1,0,-1] \text{and } \overrightarrow{X_3} = [-1,2,3],$$

use the Gram-Schmidt process to transform S to an orthonormal basis for $\mathbb{R}^3$.

(10 marks)

(b) Determine Im $[(1+2i)(2+3i)]$. (2 marks)

(c) Find the Fourier series for the function

$$f(x) = \begin{cases} 0, -\pi \leq x \leq 0 \\ x, 0 \leq x \leq \pi \end{cases}$$

(8 marks)