JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

3RD YEAR 1ST SEMESTER 2022/23

MAIN SPECIAL

COURSE CODE: SPB 9309

COURSE TITLE: CLASSICAL MECHANICS

EXAM VENUE: **STREAM: (BED SCI)**

DATE: **EXAM SESSION:**

TIME: 2:00HRS

Instructions:

1. **Answer question 1 (Compulsory) in Section A and ANY other 2 questions in Section B.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

Useful constants

gravitational acceleration, g, = 9.8 m/s^2
velocity of light in free space = 3.0×10^8 m/s

Question 1

(a) Define the angular momentum L about a point and the torque N about the same point, hence obtain the relationship between the two. (4 Marks)

(b) **(i)** What is a conservative force field? (1 Mark)

(ii) Give a necessary and sufficient condition for a force to be conservative.

(1 Mark)

(c) Due to a force field, a particle of mass 4 kg moves along a space curve whose position vector is given as a function of time by

$$\vec{r} = (3t^2 + t)\hat{i} + (2t^4 - t^2 + 6)\hat{j} - 12t^2\hat{k}$$

Find:

(i) the velocity

(ii) the momentum

(iii) the acceleration

(iv) the force field

at any time t.
(7 Marks)

(d) A system of many particles, acted upon by several forces, changes from configuration 1 to configuration 2. Derive an expression for the work done by all the forces, hence deduce the kinetic energy for a given configuration. (5 Marks)

(e) The two masses in an Atwood machine are m_1 and m_2 (see Fig.1).

(i) Determine the acceleration of the system, a, in terms of m_1 and m_2 and state any assumption made.
(5 Marks)

(ii) Given that m_1= 2.4 kg and m_2= 2.2 kg, determine the tension in the string.
(4 Marks)

(f) An observer on Earth sees a spaceship at an altitude of 4350 km moving towards the Earth with a speed of 0.970c, where c is the speed of light in free space. Find the distance from the space ship to the Earth as measured by the captain of the spaceship.
(3 Marks)

Question 2

(a) Derive the Hamilton's equations. (10 Marks)

(b) Use Hamilton's equations to obtain the equations of motion of a uniform heavy rod of mass m and length $2l$ turning about one end which is fixed.

(10 Marks)

Question 3

(a) Muons have a mean lifetime of 2.2×10^{-6} s when at rest. They are produced at an altitude of 10 km and travel at 0.995*c* toward the earth. Find:

(i) The mean lifetime measured on earth.

(ii) The time taken to reach ground level in the earth frame.

(iii) The time taken to reach ground level in the particle's frame.

(10 Marks)

(b) At age 22, twin A sets off for a distant planet P which is 20 light-years away, leaving behind twin B on earth. Twin A flies in a rocket which can attain a speed 0.95c. Immediately he reaches P, he turns back and flies to earth.

(i) Considering himself to be at rest throughout the voyage, calculate the time determined by A to have elapsed when he arrives back on earth, hence find the supposed age of B.

(ii) Calculate the time that has elapsed as calculated by B, hence the age of A.

(iii) The ages obtained by the two are different. Explain this paradox.

(10 Marks)

Question 4

(a) **(i)** State the necessary and sufficient condition for a force field to be conservative.
(2 Marks)

(ii) Show that the force field defined by

$$\vec{F} = (y^2z^3 - 6xz^2)\hat{i} + 2xyz^3\hat{j} + (3xy^2z^2 - 6x^2z)\hat{k}$$

is conservative.
(10 Marks)

(b) A particle of mass 2 kg moves in a force field dependent on time t given by

$$\vec{F} = 24t^2\hat{i} + (36t - 16)\hat{j} - 12t\hat{k}$$

If at $t = 0$ the particle is located at $\vec{r_0} = 3\hat{i} - \hat{j} + 4\hat{k}$, and has velocity $\vec{v_0} = 6\hat{i} + 15\hat{j} - 8\hat{k}$, find the position at any time t. (8 Marks)

Question 5

(a) **(i)** Write down the Lagrangian for a simple pendulum constrained to move in a single vertical plane.

(ii) Hence find the equation of motion.
(10 Marks)

(b) *State the principle of Galilean transformation and explain its limitations under relativistic conditions.*
(2 Marks)

(c) *The length of an object in its rest frame is L_o. Show that its length L when it is in motion relative to an observer, at velocity v is less than L_o.* (8 Marks)