JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

2022/2023 EXAMINATIONS

MAIN SPECIAL

COURSE CODE: SPB 9427
COURSE TITLE: QUANTUM MECHANICS II
EXAM VENUE: **STREAM: EDUCATION**
DATE: **EXAM SESSION:**
TIME: 2:00 HRS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Useful constants
In spherical coordinates,
$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 sin^2\theta}\left(\frac{\partial^2}{\partial \phi^2}\right)$$

Question 1 (30 marks)

(a) (i) State any two postulates of quantum mechanics. (2 marks)
(ii) Distinguish between the interaction picture and the Heisenberg picture. (2 marks)

(b) A spin 1/2 particle is in the state

$$\chi = \frac{1}{\sqrt{6}}\begin{pmatrix} 1+i \\ 2 \end{pmatrix}.$$

If $\hat{S}_z$ is measured, find the probability of getting $+\hbar/2$and $-\hbar/2$respectively. (3 marks)

(c) Calculate the commutation relation $\left[\hat{x}, \hat{L}_z\right]$. (2 marks)

(d) List down the state vectors $u\rangle$ and $|d\rangle$, and use them to determine the spin state transition operators of a two-state system, $\hat{S}_+$; $\hat{S}_-$. (4 marks)

(e) From the definition of angular momentum, prove the commutation relation $\left[\hat{L}_x, \hat{L}_y\right] = i\hbar\hat{L}_z$. (3 marks)

(f) The spatial wavefunction of hydrogen is given by

$$\psi_{nlm}(r, \theta, \phi) = R_{nl}Y_l^m(\theta, \phi).$$

Explain the meaning of each term in this function. (2 marks)

(g) For a spin 1/2 elementary particle, write the spin in terms of the spin-up and spin-down matrices. (4 marks)

(h) Explain the principle of perturbation theory, indicating any limitation to the use of the theory. (3 marks)

(i) From the results of the commutator relation $\left[\hat{L}_z, \hat{x}\right]$,show how the matrix elements of $\hat{y}$can be obtained from the corresponding matrix elements of $\hat{x}$. (3 marks)

(j) Give the major steps involved in using the Ritz variation method. (2 marks)

Question 2 (20 marks)

(a) The number state vector $n\rangle$ of a quantized harmonic oscillator satisfies the state transition algebraic relations

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle; \qquad \hat{a}^{+}|n\rangle=\sqrt{n+1}|n+1\rangle$$

(i) Identity the operators $\hat{a}$and $\hat{a}^{+}$. (1 mark)

(ii) Show that (4 marks)

(b) The angular momentum in spherical coordinates is given by

$$\vec{L} = -i\hbar\left(\hat{\phi}\frac{\partial}{\partial\theta} - \hat{\theta}\frac{1}{sin\theta}\frac{\partial}{\partial\phi}\right).$$

(i) Express $\hat{\theta}$ and $\hat{\phi}$ in terms of the rectangular unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$, hence obtain $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$. (5 marks)

(ii) Obtain the lowering operator $\hat{L}_-$. (3 marks)

(c) Angular momentum operators can be defined by the following matrices:

$$\hat{L}_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}; \quad \hat{L}_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i \\ 0 & -i \\ 0 \end{pmatrix}; \quad \hat{L}_z = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ 0 & -1 \end{pmatrix}.$$

Determine the commutation bracket $[\hat{L}_x, \hat{L}_y]$ in terms of $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$. (7 marks)

Question 3 (20 marks)

(a) Prove by direct matrix multiplication that the Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 \\ 0 \end{pmatrix}; \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ 0 \end{pmatrix}; \qquad \sigma_x = \begin{pmatrix} 0 \\ 0 & -1 \end{pmatrix}$$

anticommute and they follow the commutation relations $[\sigma_x, \sigma_y] = 2i\sigma_z, x, y, z$ cyclic.

(8 marks)

(b) For an electron, it is given that

$$\hat{S}^2 s, m_s\rangle = \hbar^2 s(s+1)s, m_s\rangle, \hat{S}_z s, m_s\rangle = \hbar m_s.$$

(i) State what is represented by $\hat{S}$, s and m_s. (3 marks)

(ii) Express the state of the electron in terms of the up and down spinors, $\chi_{+,}\chi_{-}$. (4 marks)

(c) Derive the Heisenberg's equation of motion. (4 marks)

Question 4 (20 marks)

(a) For the hydrogen atom, use the Schroedinger equation in spherical coordinates to show that the allowed energies are given by

$$E_n = -\left[\frac{m}{2\hbar}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\right]\frac{1}{n^2}, \quad n = 1,2,3,\ldots$$ (13 marks)

(b) A particle of charge q and mass m, which is moving in a one-dimensional harmonic potential of frequency ω, is subject to a weak electric field ε in the x-direction. Find an exact expression for its energy, and the energy to first nonzero correction, using perturbation theory. (7 marks)

Question 5 (20 marks)

(a) Discuss the differences between bosons and fermions, giving examples of each. (5 marks)

(b) Given the commutators

$$\left[\hat{L}_z, x\right] = i\hbar y, \qquad \left[\hat{L}_z, y\right] = i\hbar x, \qquad \left[\hat{L}_z, z\right] = 0,$$

use the quantum numbers n, l and m to show that the selection rule for m is $\Delta m = \pm$ 1or 0.

(7 marks)

(c) The Schroedinger equation of a particle confined to the positive x-axis is

$$\frac{-\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + mgx\Psi = E\Psi$$

where $\Psi(0) = 0, \Psi(x) \rightarrow 0$as $x \rightarrow \infty$and E is the energy eigenvalue. Use the trial function $xexp(-ax)$ to estimate the energy, and hence obtain the best value of the parameter a.

(8 marks)