JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

2021/2022 EXAMINATIONS

MAIN SPECIAL

COURSE CODE: SPH 205
COURSE TITLE: MATHEMATICAL METHODS FOR PHYSICS II
EXAM VENUE: **STREAM: EDUCATION**
DATE: **EXAM SESSION:**
TIME: 2:00 HRS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

SECTION I (30 MARKS)

Question 1 (Compulsory)

(a) Define the following terms:
   (i) Essential singularity. (2 marks)
   (ii) Power series. (1 mark)

(b) (i) State the fundamental theorem of algebra. (1 mark)
   (ii) A complex number is given as z = 3 + i. Write z in polar form. (3 marks)

(c) Evaluate $\left(-\sqrt{3}-i\right)^{-12}$ (3 marks)

(d) Find the Euclidean inner product $\langle [1, i], [0, 2i] \rangle$. (3 marks)

(e) Given the system of linear equations

$$a_1x_1 + a_2x_2 = a_3$$
$$b_1x_1 + b_2x_2 = b_3$$

obtain an expression for x_2 in the form of a ratio of two determinants. (3 marks)

(f) Evaluate f_x and f_y for the function $f(x, y, z) = x^2 y^3 + 2xyz - 3yz$ at the point (−2, 1, 2). (3 marks)

(g) Find the determinant of $A = \begin{pmatrix} 1\,2 & -3 \\ 0\,2 & -1 \\ 3\,4 & -1 \\ 0\,2 & \end{pmatrix}$. (4 marks)

(h) Find the general solution of the differential equation $x' + ax = 0$ (3 marks)

(i) Show that in a Fourier series of a function f(x) in the interval [-π, π], the Fourier coefficient a_0 is given by

$$a_0 = \frac{1}{2\pi}\int f(x)dx$$

(4 marks)

Question 2 (20 marks)

(a) Determine the partial second order derivatives of $f(x, y) = x\cos y + ye^x$, hence show that the second partial derivative is a commutative operation. (8 marks)

(b) Show that the function $f(z) = e^{-y}\cos x + ie^{-y}\sin x$ satisfies the Cauchy-Riemann conditions, hence find $f'(z)$. (8 marks)

(c) Given the Euler's formula, derive de Moivre's theorem. (4 marks)

Question 3 (20 marks)

(a) Find the linearization of $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$ at the point (3, 2). (9 marks)

(b) Investigate whether the following linear system of equations can be solved using Cramer's rule.

(7 marks)

(c) Show that the set $S = \left\{\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)\right\}$is orthonormal. (4 marks)

Question 4 (20 marks)

(a) (i) Name an eigenvalue equation in quantum mechanics. (1 mark)

(ii) Let $B = \begin{pmatrix} 1 & \\ 4 2 & \end{pmatrix}$.Find the eigenvalues of *B* and the associated eigenvectors.

(12 marks)

(b) Show that the following linear system of equations can be solved using Cramer's rule.

$$-2x_1 + 3x_2 - x_3 = 1$$
$$x_1 + 2x_2 - x_3 = 4$$
$$-2x_1 - x_2 + x_3 =- 3$$

(7 marks)

Question 5 (20 marks)

(a) Given the basis $S = \{\vec{X}_1, \vec{X}_2, \vec{X}_3\}$for $\mathbb{R}^3$,where

$$\vec{X}_1 = [1,1,1], \vec{X}_2 = [-1,0,-1]\text{and } \vec{X}_3 = [-1,2,3],$$

use the Gram-Schmidt process to transform *S* to an orthonormal basis for $\mathbb{R}^3$.

(10 marks)

(b) Determine Re [(1 + i)(2 + i)]. (2 marks)

(c) Find the Fourier series for the function

$$f(x) = \begin{cases} 0, -\pi \le x \le 0 \\ x, 0 \le x \le \pi \end{cases}$$

(8 marks)