JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTURIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF ACTUARIAL SCIENCE

3RD YEAR 1ST SEMESTER 2023/2024 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: WAB 2103

COURSE TITLE: MATHEMATICAL MODELING

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question One Compulsory (30mks)

a) Briefly describe the following terms (8mks)
   i) Order of a Difference equation
   ii) Differential equations
   iii) Non - Homogeneous difference equations
   iv) Mathematical Modelling

b) Find the order of the following Difference equations
   i) $x_{n+1}.x_{n-1} = 2x_{n-2}$ (2mks)
   ii) $x_{n+1} = x_{n-1} + 2x_n$ (2mks)
   iii) $x_{n+1} = 2x_n$ (2mks)

c) Your neighbor has just bought a new car for $29 790. When you look up a motoring magazine on secondhand car prices you find out that this particular model of car loses, on average, 17% of its value each year.

   i) Write down a difference equation to describe the decreasing value of the car each year (3mks)
   ii) What will be the secondhand value of the car after your neighbour has owned it for six years, that is, at the start of the seventh year? Give your answer correct to the nearest dollar (3mks)

d) Solve the differential equation $y' = e^{x-y}.$ (4mks)

e) Solve the following difference equations to find x_{10}

   i) $x_{n+1} = 1.8x_n + 10, x_0 = 20$ (3mks)

   ii) $x_{n+1} - x_n = 3x_n + 4, x_0 = 2$ (3mks)

Question Two (20mks)

When first investigated, a lake in a national park contained 10 000 trout. If left to natural forces, the trout numbers in the lake would increase, on average, by 20% per year. On this basis, the park authorities give permission for 1800 trout to be fished from the lake each year.

i) Write down a difference equation that determines the number of trout in the lake each year. (2mks)

ii) Under these conditions, how long will it take for trout numbers in the lake to double? (3mks)

iii) Graph trout numbers against year for 15 years. Comment on the pattern of growth.(3mks)

iv) Suppose that the park authorities had allowed 2200 trout to be fished from the lake each year. Write down a difference equation that determines the number of trout in the lake each year. (2mks)

v) Graph trout numbers against year for 15 years. Comment on the pattern of growth. (3mks)

vi) If 2200 trout are fished from the lake each year, the trout will disappear. In which year?

(3mks)

vii) Suppose the park authorities allow 2000 trout to be fished from the lake each year. Investigate (4mks)

Question Three (20mks)

A population of insects in a region will grow at a rate that is proportional to their current population. In the absence of any outside factors the population will triple in two weeks' time. On any given day there is a net migration into the area of 15 insects and 16 are eaten by the local bird population and 7 die of natural causes.

i) If there are initially 100 insects in the area will the population survive? (10mks)

ii) If not, when do they die out? (10mks)

Question Four (20mks)

a) The population of a community is known to increase at the rate proportional to the number of people present at a time t. If the population has double in 6 years, how long it will take to triple?

(5mks)

b) A bacterial culture starts with 500 bacteria and doubles in size every half-hour.

i) How many bacteria are there after 3 hours? (3mks)

ii) How many bacteria are there after t hours? (2mks)

c) Suppose that you get a 30 year mortgage at 8% interest rate per annum. How much can you afford to borrow if you can afford to make a monthly payment of $1,000 (10mks)

Question Five (20mks)

A runner in the Ndakaine Marathon starts out too fast; as consequence her speed decreases throughout the race at a rate inversely proportional to the square root of time.

(a) Set up the differential equation that describes how her speed depends on time. (2mks)

(b) The runner starts out running 8 mph, and after one hour her speed has reduced to 7 mph. Find her speed as a function of time. (10mks)

(c) Find the function that expresses her distance from the starting line as a function of time. (8mks)