JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

3RD YEAR 1st SEMESTER 2023/2024

REGULAR (MAIN)

COURSE CODE: WAB 2301

COURSE TITLE: METHODS OF ACTUARIAL INVESTIGATIONS I

EXAM VENUE: **STREAM: (BSc Actuarial Science)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE [30 marks]

a. State the three theories explaining the shape of the yield curve.

(6 marks)

b. Describe a one-factor term structure model?

(2 marks)

c. A three-month forward contract exists on a zero-coupon corporate bond with a current price per £100 nominal of £42.60. The yield available on three-month government securities is 6% *pa* effective.

Calculate the forward price.

(2 marks)

d. The current price of a share is £200. The share pays dividends continuously to provide a fixed dividend yield, and the current dividend is £5 *pa*.

Calculate the forward price of a five-year contract on one share if the risk-free force of interest is 5% *pa*.

(3 marks)

e. Suppose that the exercise price of a 3-month European call option on Share X is 100 and the continuously compounded risk-free rate of return is 12% *pa.*

Calculate the lower bound for the option's price if the current price of Share X is:

(i) 115

(ii) 125.

(4 marks)

f. Calculate the lower bound for a 3-month European put option on Share X if the current share price is 95, the exercise price is 100 and the continuously compounded risk-free rate is 12% *pa*.

(2 marks)

g. A man now aged exactly 50 has built up a savings fund of £400,000. In order to retire at age 60, he will require a fund of at least £600,000 at that time. The annual returns on the fund, i , are independent and identically distributed, with $1+ i \sim \log N(0.075,0.1^2)$.

Calculate the probability that, if the man makes no further contributions to the fund, he will be to retire at age 60.

(5 marks)

h. When valuing derivatives it is often assumed that the price of the underlying security follows a geometric Brownian motion with stochastic differential equation:

$dS_t = S_t(\mu dt + \sigma dZ_t)$

where Z_t represents a standard Brownian motion. List the 2 advantages and 2 disadvantages of this assumption.

(4 marks)

i. Explain what is meant by the continuous-time lognormal model of security prices.

(2 marks)

QUESTION TWO [20 marks]

a. The short rate of interest is governed by the stochastic differential equation (SDE):

$$dr_t = 0.6(0.04 - r_t)dt + 0.006dB_t$$

where B_t is a standard Brownian motion.

By considering the function $f(r_t,t) = r_t e^{0.6t}$, or otherwise, solve this SDE.

(10 marks)

b. A bond trader assumes that $f(t,T)$, the instantaneous forward rate of interest at time T implied by the market prices of bonds at the current time t , can be modelled by:

$$f(t,T)=0.04e^{-0.2\tau}+0.06(1-e^{-0.2\tau})+(1-e^{-0.2\tau})e^{-0.2\tau}$$

where $\tau = T-t$

i. Sketch a graph of $f(t,T)$ as a function of τ.

(3 marks)

ii. Calculate the following quantities using this model:

a) the instantaneous forward rate of interest in two years' time

b) the current price of a 10-year zero-coupon bond

c) the current 10-year spot rate.

You should express your answers to (a) and (c) as annualized continuously compounded rates.

(7 marks)

QUESTION THREE [20 marks]

a. A fixed-interest security pays coupons of 8% *pa* half-yearly in arrears and is redeemable at 110%.

Two months before the next coupon is due, an investor negotiates a forward contract to buy £60,000 nominal of the security in six months' time. The current price of the security is £80.40 per £100 nominal and the risk-free force of interest is 5% *pa*.

i. Calculate the forward price.

(2 marks)

ii. On the same day, a different investor negotiates a forward contract to purchase £50,000 nominal of the security in ten months' time.

Calculate the forward price of this contract. (4 marks)

b. The annual returns on a fund, i , are independent and identically distributed. Each year, the distribution of $1+i$ is lognormal with parameters $\mu = 0.075$ and $\sigma^2 = 0.025^2$.

Calculate the upper and lower quartiles for the accumulated value at the end of 5 years of an initial investment of £1,000. (4 marks)

c. A lump sum of $14,000 will be invested at time 0 for 4 years at an annual rate of return, i'. The rate of return, once determined, will be the same in each of the four years. $1+i'$ has a lognormal distribution with mean 1.05 and variance 0.007.

Calculate the probability that the investment will accumulate to more than $20,000 in 4 years' time.

(10 marks)

QUESTION FOUR [20 marks]

a. The returns from an investment are assumed to conform to the fixed rate model with the distribution of rates as specified below:

$$i_k = \begin{cases} 0.06 & \text{with probability } 0.2 \\ 0.08 & \text{with probability } 0.7 \\ 0.10 & \text{with probability } 0.1 \end{cases}$$

i. Calculate the expected accumulated value at the end of 5 years of an initial investment of £5,000.

(3 marks)

ii. Calculate the accumulated value at the mean rate of return.

(2 marks)

iii. Calculate the variance of the accumulated value of the investment

(5 marks)

b. Calculate the mean and variance of the accumulated value at the end of 25 years of an initial investment of £40,000, if the annual rate of return in year *k* is independent of that in any other year and i_k - *Gamma* (16,200) for all *k*.

(10 marks)

QUESTION FIVE [20 marks]

a. The shares of Abingdon Life can be modelled using a lognormal model in which the drift parameter, μ=0.104 pa and volatility, σ =0.40 pa. If the current share price is 2.00, derive a 95% confidence interval for the share price in one week's time, assuming that there are exactly 52 weeks in a year. (10marks)

b. An investor has decided to model PPB plc shares using the continuous-time lognormal model. Using historical data, the investor has estimated the annual drift and volatility parameters to be 6% and 25% respectively. PPB's current share price is $2.

i. Calculate the mean and variance of PPB's share price in one year's time.

(5marks)

ii. Calculate the probability that PPB's shares fall in value over the next year.

(5marks)