JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTURIAL SCIENCES

UNIVERSITY EXAMINATION FOR BACHELOR OF ACTUARIAL SCIENCE

2023/24

MAIN REGULAR

COURSE CODE: WAB 2307

COURSE TITLE: Financial Economics I

EXAM VENUE **STREAM: B.Sc. Actuarial Science**

DATE:…… **EXAM SESSION: ONE**

TIME: 2 HOURS

Instructions to the Candidate:

1. **Answer ONE and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question One: 30 Marks

a. Explain FOUR axioms that are required for the derivation of the expected utility theorem [4 Marks]

b. Consider two risky assets A and B with cumulative probability distribution functions:

$$F_A(W) = W$$
$$F_B(W) = W^{\frac{1}{2}}$$

And in both cases; $0 \leq w \leq 1$

i. Show that A is preferred to B on the basis of first order stochastic dominance [4 Marks]

ii. Verify explicitly that A also dominate B on the basis of second order stochastic dominance [4 Marks]

c. Nyanumba intends to invest in the two assets X and Y as shown below:

	$Asset\ X$	$Asset\ Y$
$Expected\ Return$	6%	8%
$Variance\ of\ Returns$	4%%	25%%

The correlation coefficient of the rate of return of the two assets is denoted by ρ is assumed to take the value 0.5. Nyanumba is expected to have an expected utility function of the form:

$$E_\alpha = E(r_p) - \alpha Var(r_p)$$

Where α is a positive constant and r_p is the rate of return on the asset held by Nyanumba.

i. Determine as a function of α, the portfolio that maximizes the investor expected utility. [6 Marks]

ii. Show that as α increases, Nyanumba selects and increasing portfolio of asset X [4 Marks]

d. Consider a situation where the stock price in six months from expiration date is Ksh. 42 with the exercise price of the option being Ksh.40. The risk-free rate of interest is 10% per annum and volatility is 20% per annum.
Calculate the value of:

i. European Call Option [4 Marks]

ii. European Put Option [4 Marks]

Question Two: 20 Marks

a. State THREE assumptions of Mean Variance Portfolio Theory [3 Marks]

b. Compare the returns on the investment X, Y and M where M is the market with the following probabilities:

X	Y	M	$Probabilities$
15	12	6	0.3
8	13	7	0.3
-3	-2	10	0.4

Compute:

i. The correlation between the investments. Explain the results [6 Marks]
ii. The Beta for each investment X and Y [4 Marks]
iii. The percentage of each risk for investment X and Y [4 Marks]

c. Explain the following economic characteristics and give their expression:

i. Non-Satiation [1 Mark]
ii. Risk-Aversion [1 Mark]
iii. Risk-Seeking [1 Mark]

Question Three: 20 Marks

a. A forward contract is arranged where an investor agrees to buy a share at time T for an amount K. It is proposed that the fair price for this contract at time t is;

$$f(S_t, t) = S_t - Ke^{-r(T-t)}$$

Show that this:

i. Satisfies boundary conditions [2 Marks]
ii. Satisfies the Black Scholes Partial Differential Equation (PDE) [4 Marks]

b. Let X be a diffusion process such that:

$$dX_t = \mu(t, X_t)dt + 2\sigma(t, X_t)dZ_t$$

Where Z_t is the Standard Brownian Motion, find the Standard Differential Equation for $f(t, X_t)$ using Ito's Lemma. [4 Marks]

c. Omondi has initial wealth of Ksh.100 and utility function of the form;

$$\cup(w) = log(w)$$

Where w is his wealth at any time. Investment Z offers a return of -18% or $+20\%$

With equal probability.

i. What is his expected utility if he invests nothing in Z ? [2 Marks]
ii. What is his expected utility if he invests entirely in Z? [3 Marks]
iii. What proportion of a of his wealth should he invest in investment Z to maximize his expected utility? [5 Marks]

Question Four: 20 Marks

a. Given a bond denoted by B and stock by S such that;

$$dB = rBdt,$$
$$dSt = \mu S_t dt + \sigma S_t dW_t,$$

Whereby the stock prices follow the geometric Brownian motion and r is the risk-free rate W_t is the Weiner process. We consider $f(t, S_t)$ as a derivative security whose prices depends on S and t with respect to a call function. Form a portfolio using B and S so that it behaves the same way as $f(t, S_t)$ and derive the Black Scholes Equation. [12 Marks]

b.

i. State and Explain FOUR assumptions of Black Scholes Equation [4 Marks]

ii. Highlight four parameters that affect the option prices on a non-dividend paying share. [4 Marks]

Question Five: 20 Marks

a. Explain Three categories of financial Futures. [6 Marks]

b. Consider a European Call option and European Put option with the same strike price and time to maturity. Show that they change in a value by the same amount when volatility increases from a level σ_1 to a new level σ_2 with a short period of time. [6 Marks]

c. Zero-coupon risk-free bonds are available with the following maturities and annual effective yield return:

$Maturity\ (Years)$	$Yield\ Rate$
1	0.060
2	0.065
3	0.078

Mwikali needs to buy corns for ethanol production. She wants to purchase 10,000 bushels one year from now, 1,500 bushels two years from now. The current forward prices per bushel are 3.89, 4.11 and 4.16 for one, two and three years respectively. Mwikali wants to enter to economic swap to lock these priced. Determine the sequence of payments at time one, two and three which are acceptable to Mwikali and to the corn supplier. [8 Marks]