



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE

ACTUARIAL SCIENCE

4TH YEAR 1ST SEMESTER 2023/2024

REGULAR (MAIN)

COURSE CODE: WAB 2403

COURSE TITLE: ACTUARIAL LIFE CONTIGENCIES II

EXAM VENUE:

STREAM: (BSc Actuarial Science)

DATE:

EXAM SESSION:

TIME: 2.00 HOURS

Instructions:

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a. In the context of profit-testing, explain the difference between the "profit vector" and the "profit signature".

(2 marks)

b. A certain life office sells assurance policies with term 3 years to lives aged 70. For each policy, the profit vector is estimated to be $(-50; 30; 30)$. Given that the mortality of the policyholder is expected to follow A1967-70 ultimate, calculate

i.) the profit signature per policy sold;

(3 marks)

ii.) the net present value of the profit to the office on the basis of a risk discount rate of 8% per annum.

(3 marks)

c. If a profit test for a unit-linked policy reveals negative cash flows in the second or any subsequent policy year, it is customary to eliminate these negative values by setting up sterling reserves at the end of each year.

Describe briefly the technique "zeroisation" by which these reserves are calculated.

(2 marks)

d. Define the following terms in the context of pension funds:

i.) Benefit schemes

(2 marks)

ii.) Contribution schemes

(2 marks)

e. If a life aged 35 contributes \$500 each year to his pension scheme, calculate the value of his future contributions.

(2 marks)

f. Employees contribute to a pension scheme at a rate of 6% of pensionable salary, where pensionable salary equals actual annual salary rate less \$2000 (to allow for other pension arrangements). Find the present value of the future contributions payable by a member aged exactly 30 whose current annual salary rate is \$15,500.

(4 marks)

g. A pension scheme provides each member who retires (whether for "age" or "ill-health" reasons) with an annual pension of $\frac{1}{60}$ th of his average annual income over a member's entire service, for each year of service. Fractions of years of service are included when calculating the amount of pension payable.

If contributions are paid entirely by the employer, calculate the appropriate contribution rate (as a percentage of salary) for a new entrant aged 20.

(4 marks)

h. A policy issued by a life office to a male life aged exactly 35 is subject to level weekly premiums

ceasing at exact age 65. If the man has been sick for 6 months or more when a premium falls due, the premium is waived. The policy provides the following benefits:

(a) on survival to exact age 65, an annuity of \$5,000 per annum payable monthly in advance,

(b) on death before age 65, a return of all premiums paid (including those waived during sickness) together with compound interest at 4% per annum to the date of death.

There is no waiting period and the o° periods are the same as those underlying the tables in Formulae and Tables for Actuarial Examinations. Calculate the weekly premium.

Basis: English Life Table No.12-Males, Manchester Unity Sickness Experience 1893-97,

Occupation Group AHJ, interest 4% per annum, no expenses.

(6 marks)

QUESTION TWO (20 MARKS)

A Friendly Society issues policies providing the following benefits:

- (i) A sickness benefit of \$25 per week for the first 13 weeks of sickness and \$12.50 per week thereafter, benefit ceasing at age 60. Contributions are waived during sickness.
- (ii) On death before age 60, a lump sum of \$1,000 plus a return of contributions (including any waived) without interest.
- (iii) On survival to age 60, a lump sum of \$2,000.

Contributions are payable by level weekly amounts until age 60. There is a six-month waiting period for the sickness benefit (including the premium waiver) and the o®-period may be assumed to be the same as that underlying the tables in "Formulae and Tables for Actuarial Examinations".

(a) Calculate the weekly contribution payable by a new member aged 35 on the basis given below.

(10 marks)

(b) Calculate the reserve (on the basis given below) to be held for this member five years after he joins the Society.

(10 marks)

Basis: English Life Table No. 12 - Males

Manchester Unity Sickness Experience 1893-97, Occupation Group AHJ

interest 4% per annum.

expenses are 5% of all premiums (including those waived).

QUESTION THREE (20 MARKS)

A life office issues a three-year unit-linked endowment policy to a life aged exactly 60. The annual premium is \$2,000, payable at the start of each year. The allocation proportion is 90% in year 1 and 97% thereafter. At the end of year of death during the term, the policy pays the higher of \$10,000 and the bid value of units allocated to the policy, after deduction of the fund management charge. A bonus of 2% of the (bid) value of the unit fund is payable at maturity. The life office makes the following assumptions in projecting future cash follows:

Mortality A1967-70 ultimate

Initial expenses:	\$300
Renewal expenses:	\$50, incurred at the start of the second and the third years
Fund management charge:	2% per annum, taken at the end of each year before payment of any benefits
Sterling fund interest rate:	4% per annum
Bid/offer spread:	6%
Unit fund growth rate:	10% per annum.

Construct tables to show the following:

- (i) the growth of the unit fund

(5 marks)

- (ii) the profit signature, assuming that no sterling reserves are held

(10 marks)

(iii) the profit signature after taking into account sterling reserves, given that the sterling reserves per policy are to be \$36.48 before receipt of the premium due at time 1 year and \$78.64 before receipt of the premium due at time 2 years.

In each case, indicate clearly how you calculate your table entries. Ignore the possibility of surrenders.

(5marks)

QUESTION FOUR (20 MARKS)

It is desired to set up a pension scheme for the group of employees described below. For each member there is recorded his exact age, his exact length of service with the company, and his annual rate of salary.

Age	Past Service (Years)	Rate of Salary p.a	Age	Past Service (Years)	Rate of Salary p.a	Age	Past Service (Years)	Rate of Salary p.a
25	4	8,800	35	12	12,400	45	25	15,000
25	6	8,500	35	18	12,500	45	15	14,000
25	5	8,500	35	15	12,800	45	10	13,000
25	3	8,600	35	5	12,500	45	20	13,200
25	1	8,600	35	10	12,400	45	5	13,000

The scheme will provide pensions of $\frac{1}{60}$ th of "pensionable salary" for each year of service (fractions of a year being included) and on death or withdrawal from service a return will be made of the member's contributions with 3% compound interest. All members will contribute at the same rate and the employer will contribute the same amount as each member. The contribution rate will be such as to provide exactly the benefits for future service. The basis of "Formulae and Tables" is to be used.

- i.) Assuming that pensionable salary is the average annual earnings over the three-year period ending on the retirement date, calculate the contribution rate payable by each member.

(10 marks)

- ii.) Calculate also the total liability for past service benefits. The employer wishes to meet this liability by paying additional contributions proportional to future salary payments. At what rate should these additional contributions be made?

(5 marks)

- iii.) Immediately after the scheme is set up as described above the 45-year-old member with 5 years of service withdraws. Is the position of the fund improved or worsened by this withdrawal?

(5 marks)

QUESTION FIVE (20 MARKS)

- a. A life office issues a 5-year with-profits endowment assurance policy to a life aged exactly 60. The policy has a basic sum assured of \$10,000 payable at the end of the year of death or at the maturity date. Level premiums are payable annually in advance throughout the term of the policy. Simple reversionary bonuses vest at the start of each year, including the first.

The premium is calculated according to the following basis:

mortality: A1967-70 select

interest: 4% per annum

simple reversionary bonuses at the rate of 4% per annum are assumed

initial expenses: 60% of the first premium

renewal expenses: 5% of each premium after the first

- (i) Show that the premium is equal to \$2,627.

(5 marks)

- (ii) The office holds net premium reserves using a rate of interest of 3% per annum and A1967-70 ultimate mortality.

Calculate the profit signature for this policy, assuming that the office will earn interest at 7% per annum on its assets, mortality follows the A1967-70 ultimate table, and expenses and bonuses will follow the premium basis.

(10 marks)

iii) Immediately before the fourth premium was due (before the fourth bonus declaration) the policy was made paid-up, with no entitlement to further bonuses. The paid up sum assured was 60% of the benefits immediately before the alteration, including declared bonuses. The policyholder survived to the maturity date. Interest was earned on the life funds was at 6% per annum over the period of the contract, and bonuses in the first three years followed the premium assumptions. Expenses followed the premium assumptions up to the alteration date, and no expenses were incurred after the policy was made paid-up. For each of the five years of the policy term, calculate the actual year-end profit earned on the policy.

(5 marks)