JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF ACTUARIAL SCIENCE

2023/2024 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WAB 2311

COURSE TITLE: STOCHASTIC PROCESSES 1

EXAM VENUE: **STREAM: ACTUARIAL SCIENCE**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question One Compulsory (30mks)

a) Briefly describe each of the following terms as used in stochastic processes (12mks)

i) Markov Property
ii) State Space
iii) Bernouli process
iv) Memorylessness property
v) Poisson process
vi) Transition matrix

b) On average, every one out of 10 telephones is found busy. Six telephone numbers are selected at random. Find the probability that four of them will be busy. (5mks)

c) Consider a system that can be in one of two possible states, $S = \{0 \quad 1\}$. In particular, suppose that the transition matrix is given by

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

Suppose that the system is in state 0 at time n = 0, i. e $X_0 = 0$

i) Draw the state transition diagram (2mks)

ii) Find the probability that the system is in state 0 at time n = 2 (3mks)

d) Arrivals of customers at a local supermarket are modeled by a Poisson process with a rate of (4mks)

e) λ =10 customers per minute. M is the number of customers arriving between 9:00 and 9:10, and N is the number of customers arriving between 9:30 and 9:35. What is the PMF of M +N? (4mks)

Question Two (20mks)

a) An insurance policy on an electrical device pays a benefit of 4000 if the devices fails during the first years. The amount of the benefit decreases by 1000 each successive year until it reached 0. If the device has not failed by the beginning of any given year, the probability of failure during that year is 0.4. what is the expected benefit under this policy (5mks)

b) A company prices its hurricane insurance using the following assumptions: i. In any calendar year, there can be at most one hurricane. ii. In any calendar year, the probability of a hurricane is 0.05. iii. The number of hurricanes in any calendar year is independent of the number of hurricanes in any other calendar year. Under the company's assumptions, find the probability that in a 20-year period a) there is at least one hurricane (5mks)

c) A person conducting telephone surveys must get 3 more completed surveys before their job is finished on each randomly dialed number, there is a 9% chance of reaching an adult who will complete the survey. What is the probability that the 3rd completed survey occurs on the 10th call? (5mks)

d) In modelling the number of claims filed by an individual under an automobile policy during a three year period, an actuary makes the simplifying assumption that for all integers $n \geq 0$,

$P_{n+1} = \frac{1}{5} P_n$, where P_n represents the probability that the policyholder files n claims during the period. Under this assumption, what is the probability that a policyholder files more than one claim during the period. (5mks)

Question Three (20mks)

Customers arrive in a bank according to a Poisson process with rate ¸ λ = 5 per hour. Given that the store opens at 9:00am,

i. What is the probability that exactly one customer has arrived by 9:30? (4mks)
ii. What is the probability that five have arrived by 11:30? (4mks)
iii. Given i), what is the probability that total of five have arrived by 11:30? (6mks)
iv. Given i) and ii), what is the probability that the total of 10 has arrived by the time the store closes (5:00pm)? (6mks)

Question Four (20mks)

a) A company takes out an insurance policy to cover accidents that occur at its manufacturing plant. The probability that one or more accidents will occur during any given month is 3/5. The number of accidents that occur in any given month is independent of the number of accidents that occur in all other months.
   i. Compute the Probability Mass Function (5mks)
   ii. Calculate the probability that there will be at least four consecutive months in which no accidents occur. (5mks)

b) You are visiting the rainforest, but unfortunately your insect repellent has run out. As a result, at each second, a mosquito lands on your neck with probability 0.5. If one lands, with probability 0.2 it bites you, and with probability 0.8 it never bothers you, independently of other mosquitoes.

(i) What is the expected time between successive mosquito bites? What is the variance of the time between successive mosquito bites? (5mks)

(ii) In addition, a tick lands on your neck with probability 0.1. If one lands, with probability 0.7 it bites you, and with probability 0.3, it never bothers you, independently of other ticks and mosquitoes. Now, what is expected time between successive bug bites? What is the variance of the time between successive bug bites? (5mks)

Question Five (20mks)

a) The leading brewery on the West Coast (A) has hired a TM specialist to analyze its market position. It is particularly concerned about its major competitor (B). The analyst believes that brand switching can be modeled as a Markov chain using 3 states, with states A and B representing customers drinking beer produced from the aforementioned breweries and state C representing all other brands. Data are taken monthly, and the analyst has constructed the following one-step transition probability matrix.

A	B	C
0.7	0.2	0.1
0.7	0.75	0.05
0.1	0.1	0.8

What are the steady-state market shares for the two major breweries? (10mks)

b) You are given the following information:

• Mortality for an individual can be described using a non-homogenous Markov Chain process with two states:

State 1: Alive

State 2: Deceased

You are given the following transition probability matrices for this individual:

$$Q_0 = \begin{pmatrix} 0.9 & 0.1 \\ 0 & 1 \end{pmatrix} \quad Q_1 = \begin{pmatrix} 0.8 & 0.2 \\ 0 & 1 \end{pmatrix} \quad Q_2 = \begin{pmatrix} 0.7 & 0.3 \\ 0 & 1 \end{pmatrix} \quad Q_3 = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$$

- An insurance policy is issued to this individual at time 0.
- The insured is in state 1 at the time the policy is issued.
- A benefit of $100,000 is paid out upon transition of the insured from state 1 to state 2.
- Transitions occur at the end of each time period.
- The insurance company receives a premium of $25,000 at the beginning of each time period, if the insured is in state 1 at that time.
- i = 5%

Calculate the benefit reserve for this policy at time 1, assuming the insured is in state 1 and the premium for this time period has not yet been paid. (10mks)