JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTURIAL SCIENCES

UNIVERSITY EXAMINATION FOR BACHELOR OF ACTUARIAL SCIENCE

2023/24

MAIN REGULAR

COURSE CODE: WAB 2302

COURSE TITLE: Actuarial Life Contingencies I

EXAM VENUE **STREAM: B.Sc. Actuarial Science Year Three**

DATE:…………… **EXAM SESSION: ONE**

TIME: 2 HOURS

Instructions to the Candidate:

1. **Answer ALL in-Section A and any other two questions only in Section B.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

SECTION A

QUESTION ONE: [30 MARKS]

(a). In a certain population, the force of mortality (μ_x) is given as;

	μ_x
$60 \leq x \leq 70$	0.01
$70 \leq x \leq 80$	0.015
x>80	0.025

Calculate the probability that a life aged exactly 65 will die between exact ages 80 and 83

[6 Marks]

(b). Using the survival function;

$$S(x) = 1 - \frac{x^2}{100} \quad \text{for } 0 \leq x < 10$$

i. Calculate probability of a person aged 2 surviving for the next 1 years [3 Marks]

ii. Find $P[K(4) = 1]$ [4 Marks]

iii. Find the distribution function of X [2 Marks]

(c). Show that $tP_x = e^{-\int_x^{x+t} \mu_s ds}$ [5 Marks]

(d). Show that $\mathring{e}_x = e_x + \frac{1}{2}$ [4 Marks]

(e) A husband and wife age 70 and 65 respectively effect a policy under which the benefits are:

i. A lumpsum of ksh. 100000 payables immediately on the first death
ii. A reversionary annuity of ksh. 50,000 p.a payable continuously throughout the lifetime of the surviving spouse after death of the first

Level premium is payable annually in advance till the first death occur. Calculate the annual premium on the following basis;

Male Mortality PMA 92C20

Female Mortality PMF 92C20 [6 Marks]

SECTION B: [40 MARKS]

QUESTION TWO: [20 MARKS]

(a) Show that; [5 Marks]

$$ {}_tp_{\overline{xy}} = {}_tp_{xy} + {}_tp_x(1 - {}_tp_y) + {}_tp_y(1 - {}_tp_x). $$

(b) Find an expression for the actuarial present value of a deferred annuity of $1 payable at the end of any year as long as either (20) or (25) is living after age 50 [4 Marks]

(c) Given;

$$ l_{xy} = 1000 $$

$$ l_{x+10:y} = 960 $$

$$ l_{x:y+10} = 920 $$

Calculate the probability that, of two lives aged x and y, only one will survive for the next 10years [5 Marks]

(d). Calculate the expected present value of an annuity of 6000 p.a payable annually in advance to a man aged 65, reducing to 4000 p.a continuing in payment to his wife now aged 61 if she survives him.

Basis:

Mortality a(55) ultimate male/female

Interest rate of 8%per annum, Ignore possibility of divorce. [6 Marks]

QUESTION THREE: [20 MARKS]

(a) Suppose that in a particular life table:

$$l_x = 100 - x \qquad\qquad for\ 0 \le x \le 100$$

Calculate the complete expected future lifetime of a life currently aged 50. [5 Marks]

(b) A life insurance company is considering selling with-profit endowment policies with a term of twenty years and initial sum assured of £100,000. Death benefits are payable at the end of the policy year of death. Bonuses will be added at the end of each policy year.

The company is considering three different bonus structures:

(1) simple reversionary bonuses of 4.5% per annum

(2) compound reversionary bonuses of 3.84615% per annum

(3) super compound bonuses where the original sum assured receives a bonus of 3% each year and all previous bonuses receive an additional bonus of 6% each year.

(i) Calculate the amount payable at maturity under the three structures. [5Marks]

(ii) Calculate the expected present value of benefits under structure (2) for an individual aged 45 exact at the start, using the following basis:

Interest 8% per annum

Mortality AM92 Select

Expenses ignore [5 Marks]

(iii) Calculate the expected present value of benefits, using the same policy and basis as in (ii) but reflecting the following changes:

(a) Bonuses are added at the start of each policy year (the death benefit is payable at the end of the policy year of death).

(b) The death benefit is payable immediately on death (bonuses are added at the end of each policy year).

(c) The death benefit is payable immediately on death, and bonuses are added continuously. [5 Marks]

QUESTION FOUR: [20 MARKS]

(a)

If ${}_{15}P_{45} = 0.038$, $P_{45:\overline{15}|} = 0.056$ and $A_{60} = 0.625$ find $P^{1}_{45:\overline{15}|}$.

[4 Marks]

(b)

Suppose $\mu_x = 1/(110 - x)$ for $0 \le x < 110$.

Find ${}_{10}p_{20:30}$, ${}_{10}p_{\overline{20:30}}$.

[8 Marks]

(c)

Show that ${}_{n}q^{1}_{xy} = {}_{n}q^{\;2}_{xy} + {}_{n}q_x\, {}_{n}p_y$.

[4 Marks]

(d) If the probability that (35) will survive for 10 years is a and the probability that (35) will die before (45) is b, what is the probability that (35) will die within 10 years after the death of (45)? Assume the lives are independent. [4 Marks]

QUESTION FIVE: [20 MARKS]

1. (a) Assuming that both lives are independently subject to AM92 mortality, calculate the following: [6 Marks]

(i) $_3p_{45:41}$

(ii) $q_{66:65}$

(iii) $\mu_{38:30}$

(b). [6 Marks]

Given that $_nq_x = 0.3$ and $_nq_y = 0.5$, calculate $_nq_{xy}$ and $_nq_{\overline{xy}}$.

(c) William, aged 75, and Laura, aged 80, are the guardians of a child. They take out a life assurance policy that provides a payment of £25,000 immediately when the second of them dies. Level annual premiums are payable in advance whilst the policy is in force.

(i) Calculate the annual gross premium, using the basis given below. [4 Marks]

(ii) Calculate the gross premium prospective reserve just before the sixth premium is paid, using the basis given below, assuming that both William and Laura are still alive at that time. [4 Marks]

Basis: Mortality: PMA92C20 for William, PFA92C20 for Laura Interest: 4% pa effective
Expenses: Initial: £250 Renewal: 5% of each premium, excluding the firs