JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

2023/2024 EXAMINATIONS

MAIN REGULAR

COURSE CODE: SPB 9427
COURSE TITLE: QUANTUM MECHANICS II
EXAM VENUE: **STREAM: EDUCATION**
DATE: **EXAM SESSION:**
TIME: 2:00 HRS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Useful formulae
In spherical coordinates,
$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2 sin^2\theta}\left(\frac{\partial^2}{\partial\phi^2}\right)$.

Raising and lowering operators:

$$a_{\pm} = \frac{1}{\sqrt{2m}}\left(\frac{\hbar}{i}\frac{d}{dx} \pm im\omega x\right)$$

Question 1 (30 marks)

(a) An electron in the hydrogen atom is in a state where the angular momentum quantum number $l = 1$.Find the possible values of m_l,hence find the possible angles of orientation of the angular momentum vector. (3 marks)

(b) Prove the commutation identity $[\hat{A}\hat{B}, \hat{C}\hat{D}] = \hat{A}[\hat{B}, \hat{C}]\hat{D} + [\hat{A}, \hat{C}]\hat{B}\hat{D} + \hat{C}\hat{A}[\hat{B}, \hat{D}] + \hat{C}[\hat{A}, \hat{D}]\hat{B}$. (3 marks)

(c) Use the raising operator to calculate the wave function for the first excited state of the harmonic oscillator. (3 marks)

(d) Distinguish between the Schroedinger picture and the Heisenberg picture. (2 marks)

(e) For a spin 1/2 elementary particle, write the spin in terms of the spin-up and spin-down matrices. (3 marks)

(f) Use an arbitrary function $f(x)$to obtain the commutation relation between the position operator $x = \hat{x}$and the momentum operator $\hat{p}_x = -i\hbar\frac{d}{dx}$. (3 marks)

(g) Carbon has two $2p$ electrons outside the filled subshells. Find the total orbital and spin quantum numbers of carbon, and explain any forbidden combinations of S and L. (3 marks)

(h) Find the Hermitian conjugate of the matrix

$$\hat{A} = \begin{pmatrix} 1 & 2 & 3i \\ 1+i & 1 & 0 \end{pmatrix}$$ (2 marks)

(i) Explain the conditions under which the Ritz variational principle is suitable for use.

(3 marks)

(j) Discuss the differences between bosons and fermions. (2 marks)

(k) Given the commutator $[\hat{L}_z, z] = 0$,simplify $\langle n'l'm'|[\hat{L}_z, z]|nlm\rangle$hence explain the resultant selection rule. (3 marks)

Question 2 (20 marks)

(a) The number state vector $n\rangle$ of a quantized harmonic oscillator satisfies the state transition algebraic relations

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle; \qquad \hat{a}^{+}|n\rangle = \sqrt{n+1}|n+1\rangle$$

(i) Identity the operators $\hat{a}$and $\hat{a}^{+}$,hence express the harmonic oscillator operator in terms of the number operator. (6 mark)

(ii) Show that $[\hat{a}, \hat{a}^{+}] = 1$. (4 marks)

(b) Use an arbitrary function *f* to find the products $a_- a_+$ and $a_+ a_-$ of the raising and lowering operators, hence obtain $a_- a_+ - a_+ a_-$. (10 marks)

Question 3 (20 marks)

(a) Prove by direct matrix multiplication that the Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \qquad \sigma_x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

anticommute and they follow the commutation relations $[\sigma_x, \sigma_y] = 2i\sigma_z, x, y, z$cyclic.

(8 marks)

(b) A linear harmonic oscillator is in the excited state

$$\psi_1(x) = \left(\frac{\alpha}{2\sqrt{\pi}}\right)^{1/2} 2\alpha x exp\left(\frac{-\alpha^2 x^2}{2}\right).$$

Determine $\langle\psi_1|\psi_1\rangle$,hence obtain the value of *x* for which $\langle\psi_1|\psi_1\rangle$will be maximum.

(4 marks)

(c) (i) Derive the Heisenberg's equation of motion. (4 marks)

(ii) Given the Hamiltonian of a simple harmonic oscillator as

$$\hat{H} = \frac{\hat{p}}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2,$$

find the equation of motion of the operator $\hat{x}$in the Heisenberg picture. (4 marks)

Question 4 (20 marks)

(a) (i) For the hydrogen atom, show that the Schroedinger equation in spherical coordinates can be written in the form,

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \left[V + \frac{\hbar^2}{2m} + \frac{l(l+1)}{r^2}\right] u = Eu$$

where $u(r) = rR(r)$. (10 marks)

(ii) A hydrogen atom is in a state where $l = 3$.For this state, determine the allowed values of m_l and the allowed angles of orientation of$\vec{L}$ relative to the *z*-axis, where m_l and$\vec{L}$ have their usual meanings. (3 marks)

(b) A particle of charge q and mass m, which is moving in a one-dimensional harmonic potential of frequency ω, is subject to a weak electric field ε in the x-direction. Find an exact expression for its energy, and the energy to first nonzero correction, using perturbation theory. (7 marks)

Question 5 (20 marks)

(a) (i) Explain the conditions under which the Ritz variational principle is suitable for use.

(2 marks)

(ii) The Schroedinger equation of a particle confined to the positive x-axis is

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + mgx\Psi = E\Psi$$

where $\Psi(0) = 0, \Psi(x) \rightarrow 0$as $x \rightarrow \infty$and E is the energy eigenvalue. Use the Ritz variational method with the trial function x exp(-ax) to estimate the energy, and hence obtain the best value of the parameter a. (10 marks)

(b) i. Explain the physical significance of a commutation relation between two operators.

(2 marks)

ii. State the canonical commutation relations for a particle moving in three dimensions.

(1 mark)

iii. Hence calculate the commutation relationship $[\hat{L}_y, \hat{L}_z]$. (5 marks)