JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE

1ST YEAR 2ND SEMESTER 2023/2024 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WMB9105

COURSE TITLE: LINEAR ALGEBRA 1

EXAM VENUE: **STREAM: BED AND ACT SCIENCE**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) Given vectors $u = (9,\ 3, -4,\ 0, 1)$ and $v = (0, -3,\ 2, -1, 7)$. Compute

i.) $u - 4v$ (2marks)

ii.) $(v.u)$ (2marks)

iii.) $||u||$and $||v||$ (2marks)

b) Compute $3\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}^T - 2\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ (4marks)

c) i) State Cauchy- Schwarz inequality (2marks)

ii) Given vectors $a = (4, -5)$ and $b = (1, -1)$ verify the Cauchy-Schwarz Inequality (3marks)

d) Determine the volume of a parallelepiped given by the vectors $u = (2j + 3k)$, $v = (5i + 6j + k)$ and $w = (3i + 4j + 5k)$ (5marks)

e) Let $T\colon\ \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation. Find $T\begin{bmatrix} 8 \\ 3 \\ 7 \end{bmatrix}$ if $T\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and

$T\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ (5marks)

f) Show that $\mathbb{R}^3$ is spanned by $\{(1, 0, 1), (1, 1, 0), (0, 1, 1)\}$ (5marks)

QUESTION TWO (20 MARKS)

a) Consider the triangle with vertices $A(2, 0, -3), B(5, -2, 1)$ and $C(7, 5, 3)$

i) Show that it is a right-angled triangle. (6marks)

ii) Find the lengths of the three sides and verify the Pythagorean theorem (4marks)

b) Let $T\colon \mathbb{R}^3 \to\ \mathbb{R}^2$ be a transformation defined by $T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ for all $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \epsilon\ \mathbb{R}^3$

Show that T is a linear transformation (10marks)

QUESTION THREE (20 MARKS)

a) Given vectors p= $(1, 2, 3)$ and $q = (3, 4, 5)$. Find

i) The angle between p and q (3marks)

ii) Projection p onto q (3marks)

b) i) Define a vector space (2marks)

ii) List two examples of vector spaces (2marks)

iii) Show that the set of all polynomials of degree at most 2 forms a vector space. (5marks)

c) Verify $A^2 - A - 6I = 0$ if $A = \begin{bmatrix} 3 & -1 \\ 0 & -2 \end{bmatrix}$ (5marks)

QUESTION FOUR (20 MARKS)

a) Let U be the set of all points (x, y) from $\mathbb{R}^2$ in which $x \geq 0$. Determine if $U$is a subspace of $\mathbb{R}^2$. (5marks)

b) Find the an equation to the plane through $P_o(3, 0, -1)$ that is parallel to the plane with equation $2x - y + z = 3$ (5marks)

c) i) Distinguish between linearly dependent and linearly independent vectors in a Vectors space V. (4marks)

ii) Determine whether the following vectors are linearly dependent or linearly Independent in $\mathbb{R}^3\{(2, -2, 4), (3, -5, 4), (0, 1, 1)\}$ (6marks)

QUESTION FIVE (20 MARKS)

a) Consider the vector space V over the field $\mathbb{K}$ defined by $V = span\ \{(1,2,3), (4,5,6)\}$. Determine the dimension of V and provide a basis for V. (10marks)

b) Discuss the significance of linearly independent vectors in real world by providing examples from various fields, and explaining how the concept of linear independence is crucial in solving practical problems. (10marks)