JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE (WATER AND NATURAL RESOURCE PLANNING)

1ST YEAR 2ND SEMESTER 2023/2024 ACADEMIC YEAR

(ONLINE)

COURSE CODE: WMB9102

COURSE TITLE: MATHEMATICS II

EXAM VENUE: **STREAM: BSC. WATER AND NATURAL RESOURCE PLANNING**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) Find the equation of the line with the gradient 3 which passes through the points (2, -4). (3 marks)

b) A(0, 1), B(1, 4) C(4, 3) and D(3, 0) are vertices of quadrilateral ABCD. (6 marks)

i) Find the equation of diagonals AC and BD.
ii) Show that the diagonal bisect each other at right angles.
iii) Find the length of BC.

c) Find $\frac{dy}{dx}$ for $y = \sqrt{x} - \frac{8}{x^2}$ and the gradient of the curve at the point (4, 1.5). (5 marks)

d) The line joining P (3, -4) and Q(q, 0) has a gradient of 2. Find q. (3 marks)

e) Find $5A - \frac{1}{2}B$ if $A = \begin{bmatrix} 4 & 1 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & -20 \\ 18 & 8 \end{bmatrix}$ and hence the inverse of the resulting output of $5A - \frac{1}{2}B$. (4 marks)

f) Solve the system of equations using matrices (Gauss-Jordan row operations):
$2x + 3y = 8$
$3x + 2y = 7$
(4 marks)

g) Evaluate

i) $\lim_{x \to 3} (x^2 + x + 6)$ (2marks)

ii) Evaluate $\lim_{x \to \infty} \frac{3x+5}{x^2+6x+3}$ (3marks)

QUESTION TWO (20 MARKS)

a) Find the second derivative for the function $2x^3 - 3y^2 = 8$. (6 marks)
b) State and sketch any three types of discontinuous functions. (6 marks)
c) Find the equation of the tangent to the curve $y = x^2 + 3x + 2$ at the point (2, 12). (4 marks)

d) Given two matrices $A = \begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 8 \end{bmatrix}$. Find (4 marks)

i) B^{-1}
ii) AB^{-1}

QUESTION THREE (20 MARKS)

a) Consider the curve $3x^2 - xy + 4y^2 = 141$.

i. Find the formula in x and y for the slope of the tangent line at any line (x, y) of the curve. (5 marks)

ii. Write the slope intercept equation of the tangent to the curve at the point (1, 6). (2 marks)

iii. Find the coordinates of all other points on the slope of the tangent line is same as the slope of the tangent line at (1, 6). (3 marks)

b) Show that the slope of a curve tangent to the curve $y = \frac{1}{(1-2x)^3}$ is positive and hence find the equation of the tangent to the curve at (1, 1). (5 marks)

c) Use Cramer's rule to solve the system $\begin{array}{l} 3x - 2y = 4 \\ 6x + y = 13 \end{array}$ (5 marks)

QUESTION FOUR (20 MARKS)

a) Find the area between the curve and x-axis for the function $y = x^2 + 3x$ between x=-1 and x=2. (5 marks)

b) Given $\frac{dy}{dx} = 3x^2 + 4x + 3$. Find

i) the general solution for the differential equation (3 marks)

ii) the equation of the gradient function which passes through (1, 10). (3 marks)

c) Evaluate $\int_4^9 x^{\frac{3}{2}}\, dx$ (4 marks)

d) Find the derivative of $y = (x^2 - 2)^4$ and hence the equation of the tangent at (1, 1).

(5 marks)

QUESTION FIVE (20 MARKS)

a) Use matrix adjoint rule to solve the following system of linear equations

$5x - 3y - 10z = -9.$

$2x + 2y - 3z = 4.$

$-3x - y + 5z = -1.$ (8marks)

b) A stone is thrown in the air. Its height s at any time t, in seconds is given by $s = 23 - t - 12t^3$. Determine

i) The distance travelled after 5 seconds. (2marks)

ii) The velocity at t=2 seconds. (2marks)

iii) The maximum height) attained? (3marks)

c) Use the quotient rule to find the derivative of the following

$y = \frac{2x^2+1}{3-2x}$ (5marks