JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF   SCIENCE (AGED, BIOLOGICAL SCI., COMMUNITY HEALTH, PUBLIC HEALTH, ENGINEERING)
1ST   YEAR 2ND SEMESTER 2023/2024 ACADEMIC YEAR
(MAIN/SIAYA/KISUMU)

COURSE CODE: WMB9102

COURSE TITLE: MATHEMATICS II

EXAM VENUE: STREAM: BSC. COMMUNITY HEALTH, PUBLIC HEALTH, ENGINEERING, AGED

DATE: EXAM SESSION:

TIME: 2.00 HOURS

Instructions:

1. Answer question one (compulsory) and any other two questions.

2.   Candidates are advised not to write on the question paper.

3. Candidates must hand in their answer booklets to the invigilator while in the examination room.

QUESTION ONE (30 MARKS)

a) Find the equation of the perpendicular bisector of the line joining P(-4, 5) to Q(2, 3).

(5 marks)

b) The line with equation $5x + y = 20$ meets x-axis at A and the line with $x + 2y = 22$ meet y-axis at B. The two lines intersect at point C. Calculate the coordinates of A, B and C.

(5 marks)

c) The points X(2, -1), Y(8, y) and Z(11, 2) are collinear. Find the value of y. (3 marks)

d) Given $A = \begin{bmatrix} 2 & 22 \\ 3 & 45 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$. Find $A^T B^T$ (3 marks)

e) Solve the system of equations using matrices (Gauss-Jordan row operations):
$$x + 4y = 16$$
$$3x + 5y = 20$$

(4 marks)

f) Determine the points of discontinuity of the following function.
$$f(x) = \frac{1}{x^2 - 16}.$$

(5 marks)

g) Determine the solution to the following ordinary differential equation.
$$9xy' + 5y = 3x.$$

(5 marks)

QUESTION TWO (20 MARKS)

a) Show that the point (2, 4) lies on the curve $x^3 + y^3 - 9xy = 0$. Find the equation of the line tangent and the normal at this point. (10 marks)

b) Sketch and explain the continuity of the function $f(x) = \begin{cases} 1; & x>0 \\ 1; & x<0 \end{cases}$ (5 marks)

c) If $f(x) = \frac{1}{5}x^4 - 3x^3 + 4x$. Find $f''(x)$ and hence find $f''(-3)$. (5 marks)

QUESTION THREE (20 MARKS)

a) Given the matrix $A = \begin{bmatrix} 2 & 0 & 3 \\ -1 & 4 & -2 \\ 1 & -3 & 5 \end{bmatrix}$. Compute the determinant of A and hence the inverse of A. (7 marks)

b) Evaluate $\lim_{x \to 1} \frac{x^4 + x^2 - 1}{x^2 + 5}$ ` (3 marks)

c) Find the points of the curve with $y = x^3 + 6x^2 + 5$ where the value of the gradient is -9.

(5 marks)

d)   Find the coordinates of the point on the curve $y = x^2 + 3x - 1$ at which is parallel to the line $y = 5x - 1$.   (5 marks)

QUESTION FOUR (20 MARKS)

a)   Solve the following system using Cramer's rule
$$x + 3y + z = -2$$
$$2x + 5y + z = -5$$
$$x + 2y + 3z = 6$$
                    (7 marks)

b)   Evaluate $\int (2x^3 - 3x + 4)dx$ (3 marks)

c)   Find the derivative of $y = (2x - 1)^4$ and hence the equation of the tangent at $(2, 1)$.
                    (5marks)

e)   A stone is projected vertically upwards with a speed of 30m/s. Its length above the ground after t-seconds (t<6) is given by $h = 30t - 5t^2$. Find the maximum height reached and $\frac{d^2h}{dt^2}$.   (5 marks)

QUESTION FIVE (20 MARKS)

a)   The curve $y = x^3 - 6x^2 + 11x - 6$ cuts the x-axis at x=1, x=2 and x=3. Show that the tangents to the curve at this points at which it cuts x-axis are parallel.   (6 marks)

b)   Solve the given system of equations   (10 marks)
$$2x_1 + 7x_2 + 3x_3 = 7$$
$$x_1 + 2x_2 + x_3 = 2$$
$$x_1 + 5x_2 + 2x_3 = 5$$
By expressing the system in the form of matrix equation $AB = C$, where A is a 3×3 matrix of coefficients of the variables, B and C are suitable column matrices.

c)   Differentiate the following functions with respect to x:

$y = \cos (4x^2 - 5)$.   (4marks)