JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTURIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF ……..

2ND YEAR 2ND SEMESTER 2023/2024 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: WMB 9205

COURSE TITLE: VECTOR ANALYSIS

EXAM VENUE: **STREAM:**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 marks)

a) Show geometrically that addition of vectors is associative i.e. $\underset{\sim}{P}+\left(\underset{\sim}{Q}+\underset{\sim}{R}\right)=\left(\underset{\sim}{P}+\underset{\sim}{Q}\right)+\underset{\sim}{R}$ (5 marks)

b) Find the unit vector $\hat{a}$ in the direction of the vector $\vec{A}=2\vec{B}+3\vec{C}$ if $\vec{B}=2\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{C}=\hat{i}+4\hat{j}+\hat{k}$ (5 marks)

c) Find the angle θ between the vectors $\vec{U}=3\hat{i}+2\hat{j}+6\hat{k}$ and $\vec{V}=2\hat{i}-\hat{j}+2\hat{k}$ (5 marks)

d) Prove that the area of a parallelogram with sides $\vec{A}$ and $\vec{B}$ is $\left|\vec{A}\times\vec{B}\right|$ (5 marks)

e) If $\vec{A}=xz^3\hat{i}+2x^2yz\hat{j}+2yz^4\hat{k}$, find Curl $\vec{A}$ at $(1,-1,1)$ (5 marks)

f) If $\vec{A}=2yz\hat{i}-x^2y\hat{j}+xz^2\hat{k}$ and $\phi=2x^2yz^3$, find $\left(\vec{A}\cdot\nabla\right)\varphi$ (5 marks)

QUESTION TWO (20 marks)

a) Given that $\vec{A}=A_1\hat{i}+A_2\hat{j}+A_3\hat{k}$ and $\vec{B}=B_1\hat{i}+B_2\hat{j}+B_3\hat{k}$.

Show that $\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_1 & A_2 & A_3\\ B_1 & B_2 & B_3\end{vmatrix}$ (6 marks)

b) Find the projection of the vector $\vec{A}=4\hat{i}-4\hat{j}+7\hat{k}$ on the vector $\vec{B}=\hat{i}-2\hat{j}+\hat{k}$ (4 marks)

c) Prove that $\nabla\times\left(\nabla\times\vec{A}\right)=-\nabla^2\vec{A}+\nabla\left(\nabla\cdot\vec{A}\right)$ where $\vec{A}=A_1\hat{i}+A_2\hat{j}+A_3\hat{k}$ (10 marks)

QUESTION THREE (20 marks)

a) Find grad ϕ at the point $(1,-2,1)$ if $\phi(x,y.z)=3x^2y-y^3z^2$

b) Given $\phi(x,y.z)=x^2yz^3$ and $\vec{A}=xz\hat{i}-xy^2\hat{j}+yz^2\hat{k}$, find $\dfrac{\partial^2(\varphi\vec{A})}{\partial x\partial z}$ at the point $(1,-1,1)$ (5 marks)

c) Evaluate $\left(2\hat{i}-3\hat{j}\right)\cdot\left[\ \left(\hat{i}+\hat{j}-\hat{k}\right)\times\left(3\hat{i}-\hat{k}\right)\ \right]$ (4 marks)

a) If $\vec{A}=10x^3yz\vec{i}-6xz^3\vec{j}+4xz^2\vec{k}$ and $\vec{B}=8z\vec{i}+3y\vec{j}-7x^2\vec{k}$, Find $\dfrac{\partial^2}{\partial x\partial y}\left(\vec{A}\times\vec{B}\right)$ at $(3,5,2)$(6 marks)

QUESTION FOUR (20 marks)

a) Find a unit vector perpendicular to the plane of $\vec{i}-2\vec{j}+3\vec{k}$ and $3\vec{i}+\vec{j}+2\vec{k}$, (6 marks)

b) If $\vec{F}=(2y+3)\underset{\sim}{i}+xz\underset{\sim}{j}+(yz-x)\underset{\sim}{k}$, evaluate $\int_C \underset{\sim}{F}.d\underset{\sim}{r}$ along the following paths c:

(i) $x=2t^2$, $y=t$, $z=t^3$ from $t-0$ to $t=2$ (4 marks)

(ii) the straight lines from $(0,0,0)$ to $(0,0,1)$, then to $(0,1,1)$ and then to $(2,1,1)$ (6 marks)

(iii) the straight line joining $(0,0,0)$ and $(2,1,1)$ (4 marks)

QUESTION FIVE (20 marks)

a) Evaluate $\iint_S \vec{A}\cdot\hat{n}ds$, where $\vec{A}=18z\hat{i}-12\hat{j}+3y\hat{k}$ and S is that part of the plane $2x+3y+6z=12$ which is located in the first octant. (10 marks)

b) Verify stokes' theorem for $\vec{A}=(2x-y)\hat{i}-yz^2\hat{j}-y^2z\hat{k}$, where S is the upper half surface of the sphere $x^2+y^2+z^2=1$ and C is its boundary. (10 marks)