JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL PHYSICAL MATHEMATICS AND ACTUARIAL SCIENCE (SBPMAS)

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION SCIENCE/ARTS/SPECIAL EDUCATION
3RD YEAR 2ND SEMESTER 2023/2024 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WMB 9302
COURSE TITLE: COMPLEX ANALYSIS

EXAM VENUE: **STREAM: BED SCIENCE Y3S2**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY) – 30 MARKS

a) Define each of the following terms as used in complex analysis
   i) A circle
   ii) Deleted neighbourhood (4 marks)

b) Write $\frac{i}{\sqrt{3}+i}$ in exponential form using the principal argument (6 marks)

c) Determine the value of $f(z) = z\bar{z}^2$ at the point $1+i$ giving your answer in the form $a+bi$ (4 marks)

d) Sketch the disk represented by $0 < |Z - 1 + 2i| \leq 2$ hence
   i) State the deleted neighbourhood (4 marks)
   ii) State any two boundary points (2 mark)

iii) State any two points in the neighbourhood of the disk (2 mark)

e) Describe all the transformations represented by a complex mapping $f(z) = (1+i)z - 3 - 2i$ (4 marks)

f) Show that the function $f(x) = 3x^2y^2 - 6ix^2y^2$ is not analytic at any point but differentiable only along the coordinate axes (4 marks)

QUESTION TWO (20 MARKS)

a) Find the value of $\left(1-\sqrt{3}i\right)^i$ (5 marks)

b) Given two complex numbers $z_1 = 3\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ and $z_2 = 2\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$ determine

i) $\bar{z}_1\bar{z}_2$. ii) $\frac{z_1}{z_2}$ (5 marks)

c) Solve the compex quadratic equation $z^2 - (1+i)z + 6 - 17i = 0$ (5 marks)

d) Find the image of a line *y* = -2 under the complex mapping $w = z^2$ for *w*, $z \in \mathbb{C}$, hence sketch the line and its image under the mapping (5 marks)

QUESTION THREE (20 MARKS)

a) Determine the expression for the n[th] root of -1 hence determine $(-1)^{\frac{1}{4}}$ and sketch an appropriate circle indicating the roots w_0, w_1, w_2. and w_3(5 marks)

b) Use L'Hopital's rule to compute

$$\lim_{z \to 1+\sqrt{3}i} \frac{z^2 - 2z + 4}{z - 1 - \sqrt{3}i}$$ (5 marks)

c) Given the complex function $f(z) = u(x, y) + iv(x, y)$, verify that the function $u(x, y) = x^2 - y^2$ is harmonic hence find $v(x, y)$ the harmonic conjugate *u*,Hence find the corresponding analytic function $f(z) = u + iv$, such that $f(1+i) = 3 - 2i$ (6 marks)

d) Compute the principal value of the complex logarithm $\ln z$ for $z = \sqrt{2} + \sqrt{2}i$ (4 marks)

QUESTION FOUR (20 MARKS)

a) Use De-Moivre's theorem to evaluate $\left(1-\sqrt{3}i\right)^4$, giving your answer in the form $a+bi$, $a,b \in \mathbb{R}$ (5 marks)

b) Evaluate the line integral $I = \oint_c \left(x^2 dx - y dy\right)$ where C comprises the triangle $O(1,0)$, $A(1,1)$ and $C(0,0)$ (5 marks)

c) Shoe that the function $f(z) = \dfrac{z^2+1}{z+i}$, is discontinuos at the point $z_0 = -i$ (5 marks)

d) Use the definition of the derivative of a complex function to determine the derivative of $f(z) = 4z^2 + z$ in the region where the derivative exists. (5 marks)

QUESTION FIVE (20 MARKS)

a) Prove that if a complex function $f(z) = u(x,y) + iv(x,y)$ is analytic at any point z, and in the domain D,then the Laplace's equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$, can be verified. (6 marks)

b) Find the derivative of $\left(z^2 - (2+i)z\right)^2$ (4 marks)

c) Solve for w, given the complex function $e^w = 4+i$ for $w, \in \mathbb{C}$. (6 marks)

d) Evaluate the integral $\oint_c \dfrac{z}{z^2+16} dz$, where C is the circle $|z-2i| = 4$ using the Cauchy's integral formular. (4 marks)