JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL PHYSICAL MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE(ACTUARIAL SCIENCE WITH IT)
3RD YEAR 2NDSEMESTER 2023/2024 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WMB 9304
COURSE TITLE: OPERATIONS RESEARCH I

EXAM VENUE: **STREAM: BED SCIENCE Y3S2**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE(30 MARKS)

a) Define the following terms used in linear programming
   i) Linear inequality
   ii) Slack variable
   iii) Transport problem (6 marks)

b) State and briefly explain any three methods used in solving linear programming problems. (6 marks)

c) Use Gauss Jordan method to solve the set of simultaneous equations

$$x_1 + 2x_2 = 4$$
$$2x_2 - x_3 = -1$$
$$3x_1 - 2x_3 = 0$$

(6 marks)

d) A soft drink manufacturing company has 300ml and 150ml canned cola as its product with a profit margin of shs 8 and shs 4 per unit respectively. Both products have to undergo processes in three types of machines. The table below indicates the time required on each machine and available machine hours per week.

Requirements	Cola 300ml	Cola 150ml	Available Machine hours per week
Machine 1	3	2	300
Machine 2	2	4	480
Machine 3	5	7	560

(i) Formulate a linear programming problem aimed at maximising profit. (4 marks)

(ii) State any four suitable methods that can be used in optimization of the problem in (i) above. (4 marks)

(iii) Form a dual of the primal problem in (i) above. (4 marks)

QUESTION TWO (20 MARKS)

a) Mradi perfume company produces both perfumes and body spray from two flower extracts F_1 and F_2, the following data is provided.

	Perfume	Body Spray	Daily availability(litres)
Extract F_1	8	4	20
Extract F_2	2	3	8
Profit per litre(ksh)	70	50	

The maximum daily demand of body spray is 20 bottles of 100ml each. A market survey indicates that the daily demand of body spray wants cannot exceed that of perfume by more than 2 litres. The company wants to find out the optimal mix of perfume and body spray that maximizes the total daily profit

i) Develop a linear programming model based on the information above.

ii) Using graphical method and an isoprofit (search) line advise the firm on the number of bottles of each product to produce in order to maximize profit. (12 marks)

b) Use the Dual simplex method to solve the following LP problem

Minimize $Z = x_1 + 2x_2 + 3x_3$

Subject to

$$2x_1 - x_2 + x_3 \geq 4$$
$$x_1 + x_2 + 2x_3 \leq 8$$
$$x_2 - x_3 \geq 2$$
$$x_1, x_2, x_3 \geq 0$$

(8 marks)

QUESTION THREE(20 MARKS)

a) Use the big M simplex technique to solve the linear programming problem

Maximise $Z = 11x_1 + 15x_2$

Subject to

$$3x_1 + 5x_2 \leq 130$$
$$-4x_1 + 5x_2 \geq 25$$
$$x_1 + 5x_2 \geq 75$$
$$x_1, x_2 \geq 0$$

(10 marks)

b) Suppose in a) above the left hand side values changed from 130 to 100, 25 to 30, and 75 to 80 respectively, determine the new level of optimum production. (6 marks)

c) What would happen if the constraint $2x_1 + x_2 \leq 40$ is added to the LP problem. (4 marks)

QUESTION FOUR (20 MARKS)

a) Use the Two-phase method to solve the following LP problem

Minimize $Z = 12x_1 + 14x_2 + 15x_3$

Subject to

$$2x_1 + 4x_2 + 3x_3 \geq 32$$
$$x_1 + 2x_2 + 6x_3 \geq 32 \qquad \text{(8 marks)}$$
$$x_1, x_2 \geq 0$$

b) A company has three ware houses A, B and C and four stores W, X, Y and Z. The warehouses have altogether a surplus of 1600 units of a given commodity as follows

A	500
B	900
C	400

The four stores together need a total of 1600 units of the commodity as follows

W	400
X	700
Y	500
Z	200

The cost of transporting one unit in Ksh from each warehouse to store is shown in the table below

	W	X	Y	Z
A	400	200	600	400
B	500	600	900	200
C	350	800	500	600

Determine which method would be cheaper and by how much between Vogel's approximation and the least cost cell method as far as the cost of transport is concerned (12 marks)

QUESTION FIVE (20 MARKS)

a) Consider the linear programming problem

Maximise $Z = c_1x_1 + c_2x_2 + c_3x_3$

Subject to

$$\begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{pmatrix} x_1 + \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{pmatrix} x_2 + \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{pmatrix} x_3 + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} s_1 + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} s_2 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} s_3 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} s_4 = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

$x_1, \ldots\ldots\ldots s_4 \geq 0$

In the process of solving the problem, a tableau appears as follows

Basis	x_1	x_2	x_3	S_1	S_2	S_3	S_4	B
Z	0	-140	0	100	0	0	160	35400
x_3	0	$-\frac{1}{2}$	1	1	0	0	$-\frac{1}{2}$	18
S_2	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	1	0	0	11
S_3	0	2	0	$-\frac{1}{2}$	0	1	$-\frac{1}{2}$	16
x_1	1	1	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	31

i) Determine the solution to the linear programming problem

ii) Find the values of

$$\begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{pmatrix}, \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \\ a_{42} \end{pmatrix}, \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \\ a_{42} \end{pmatrix} and \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

iii) Find the values of C_1 , C_2 and C_3 (14 marks)

b) Form a dual of the primal problem in a) above given an additional constraint as $2x_1 + x_2 - 4x_3 = 120$ (6 marks)