JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL SCIENCES/ BED (SCI OR ARTS)

4^{th} YEAR 2^{nd} SEMESTER 2023/2024 ACADEMIC YEAR

REGULAR MAIN

COURSE CODE: WMB 9404

COURSE TITLE: FOURIER ANALYSIS

EXAM VENUE: **STREAM: (BSc. Actuarial Sci /BED (SCI OR ARTS)**

DATE: **EXAM SESSION: TIME: 2.00 HOURS**

Instructions:

1. Answer question 1 (Compulsory) and ANY other 2 questions

2. Candidates are advised not to write on the question paper.

3. Candidates must hand in their answer booklets to the invigilator while in the examination room.

QUESTION ONE [COMPULSORY] (30 MARKS)

a) Find the limit $\lim_{x\to 0} x\left\{\dfrac{-x^{18}+\frac{1}{2}x^{16}-\frac{1}{2}x^{241}+\frac{1}{12}x^{81}}{x^{6}}\right\}$

[6 Marks]

b) Determine explicitly if the following functions are odd, even or not ;

i) $f(x)=\cos 2x$ is on $-4\pi \le x \le 4\pi$ [5 Marks]

ii) $f(x)=\sin x$ on $-\pi \le x \le 4\pi$ [5 Marks]

iii) $f(x)=x\sin x$ on $-\pi \le x \le 4\pi$ [2 Marks]

iv) $f(x)=x\tan x$ on $-\pi \le x \le \pi$ [2 Marks]

c) Compute the Maclaurin series as far as x^6 term for the following functions

i) $x^2\left[\sin(x)\right]$ [4 Marks]

ii) $\dfrac{\cos\left(x^2\right)}{x^2}$ [3 Marks]

iii) e^{-x} [3Marks]

QUESTION TWO 2 MARKS)

a)Suppose function $f(x)$ is expressible in the Fourier series form

$f(x)=\dfrac{a_0}{2}+\sum\left[a_n\cos nx+b_n\sin nx\right],\left[-\pi,\ \pi\right]$.Describe fully what you understand by

i) period of f [2 Marks]

ii) f is periodic [2 Marks]

iii)periodic extension of f [3 Marks]

iv) Fourier coefficients of expansion of f [2 Marks]

b) Sketch of graph of four periodic extensions of f defined by $f(x)=2x^2$ on the interval $\left[-10,\ 10\right]$ [4 Marks]

c) Obtain the full solution of the ordinary differential differential equation

$y'' - 121y = \begin{cases} x & ;0<x<5 \\ -x & ;5\le x<10 \end{cases}$, [7 Marks]

QUESTION THREE (20MARKS)

Determine Solution of the heat equation satisfying $u_t = 4u_{xx}$, the condition $0 < x < 1,\ t > 0$ with the Dirichlet boundary conditions $u(t,0) = u(t,1) = 0$, $t > 0$ and initial conditions $u(0,x) = g(x) = 2x^2$, $0 \le x \le 1$ [20 Marks]

QUESTION FOUR (20MARKS)

Given real valued function $y = f(x)$ for which $f(x) = \cos x$ $\quad 0 < x < 2\pi$

$f(x) = f(x+2\pi)$

a)State period of $f(x)$ [2 marks]

b)Sketch the graph of $f(x) = \sin x$ over the interval $-6\pi < x < 6\pi$ [8 marks]

c) Find Fourier coefficients for function $f(x) = s\cos x$ and state $f(x) = \cos x$ in its Fourier series up to the first ten harmonics Deduce that the series is convergent [5 marks]

d) Find the Fourier half-range sine for $f(x) = \begin{cases} x \\ \pi + x \end{cases}$ [5 marks]

QUESTION FIVE (20MARKS)

a) Given the Fourier series for function $f(x) = \begin{cases} x; & -\pi < x < 0 \\ -x; & 0 \le x < \pi \end{cases}$

takes the expantion form , $f(x) = \frac{a_0}{2} + \sum \left[a_n \cos nx + b_n \sin nx\right], [-\pi, \pi]$

prove that;

$i)\ \frac{a_0}{2} = \frac{\pi}{2}$ [2 marks]

$ii)\ a_n = \frac{(-1)^n - 1}{n^2\pi} = (-1)^n$ [4 marks]

iii) $b_n = \frac{(-1)^n - 1}{n^2\pi} = \frac{(-1)^{n+1}}{n}$ [4marks]

c) i) Show that the functions $\sin mx, \cos mx,\ \ e^{imx}, e^{-imx}, m = 0, 1, 2, 3 \ldots$ are orthogonal functions on $[-\pi, \pi]$. [5 marks]

ii) Show that the functions $\sin mx, \cos mx,\ \ e^{imx}, e^{-imx}$, satisfy the Sturm-Lioville equation $-w''(x) = \lambda w(x), w(-\pi) = w(\pi) and\ w'(-\pi) = w'(\pi)$. [5 marks]