JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL PHYSICAL MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION ARTS AND BACHELOR OF EDUCATION SCIENCE

1ST YEAR 2ND SEMESTER 2023/2024 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WAB 2109

COURSE TITLE: INTRODUCTION TO PROBABILITY AND DISTRIBUTION THEORY

EXAM VENUE: **STREAM:**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) Outline THREE axioms of probabilities. (5 Marks)

b) If X is a random variable that is continuously uniformly distributed over $(2,9)$. Calculate the probability that;

  i. $X < 4$ (2 Marks)

  ii. $X > 6$ (2 Marks)

  iii. $3 < X < 7$ (2 Marks)

c) Let X be random variable with probability density function given by

$$f(x) = \begin{cases} cx^2 & -1 < x < 2 \\ 0 & otherwise \end{cases}$$

  i. Obtain the value of c, hence compute the expected value of $g(x) = 4x + 3$ (6 Marks)

  ii. Compute the variance of $g(x) = 4x + 3$ (4 Marks)

d) A person has three routes to get to work. The probability that he arrives on time using route A, B and C are 60%, 62% and 70% respectively. If he is equally likely to choose any of the routes and arrives at work on time, what is the probability that he chose route B. (3 Marks)

e) If a random variable X is normally distributed with mean μ and variance μ^2 and if $P(X \leq 8) = 0.95$, determine $P(4 \leq x \leq 11)$ (4 Marks)

f) In 20 independent trials, the probability of observing an outcome is 0.05 per trial. Find the probability of observing at least one such outcome. (2 Marks)

QUESTION TWO (20 MARKS)

a) Given that a random variable X has a probability density function given as

$$f(x) = \begin{cases} \dfrac{1}{b-a} & a < x < b;\ -\infty < a < b < \infty \\ 0 & otherwise \end{cases}$$

Obtain

  i. E(X) (2 Marks)

  ii. Var(X) (6 Marks)

b) If X is a discrete random variable whose probability distribution function is given by

$$f(x) = \begin{cases} \dfrac{x}{6} & x = 1,2,3 \\ 0 & otherwise \end{cases}$$

Compute

  i. $E(X)$ (2 Marks)

  ii. $Var(X)$ (4 Marks)

  iii. $E(6x^2 + 7x^3)$ (4 Marks)

  iv. $Var(3x + 4)$ (2 Marks)

QUESTION THREE (20 MARKS)

a) Consider a random variable X whose probability density function is given by

$$f(x) = \begin{cases} \frac{x}{21} & x = 1,2,3,4,5,6 \\ 0 & otherwise \end{cases}$$

Obtain

i. E(X) (3 Marks)

ii. Var(X) (5 Marks)

b) A lot containing 7 components is sampled for quality inspection. The lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector for inspection. Find the probability of the number of good components in the sample. (12 Marks)

QUESTION FOUR (20 MARKS)

a) Given a random variable X with probability distribution function given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \lambda > 0; x > 0 \\ 0 & otherwise \end{cases}$$

Obtain;

i. $E(X)$ (4 Marks)

ii. $var(X)$ (6 Marks)

b) Given that the random variable X represents the number of motor cycles that are used for sales on any working day for company A and company B with probability distribution given by

Company A

X	1	2	3
$f(x)$	0.2	0.5	0.3

Company B

X	1	2	3	4
$f(x)$	0.2	0.3	0.2	0.3

Compare the variance distribution of the probability distribution between the two companies. (10 Marks)

QUESTION FIVE (20 MARKS)

a) Using the moment generating function compute the mean and variance of a Binomial distribution. (15 Marks)

b) A random variable X represents the number of failures preceding the first success whose probability distribution function is represented as

$$f(x) = pq^x; \qquad x = 0,1,2$$

Obtain the mean of X (5 Marks)