JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL,PHYSICAL & ACTUARIAL SCS

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE

ACTUARIAL SCIENCE

4TH YEAR 2ND SEMESTER 2023/2024 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: WAB 2410

COURSE TITLE: BAYESIAN INFERENCE AND DECISION THEORY

EXAM VENUE:LAB 5 **STREAM:ACTUARIAL**

DATE: 24/4/24 **EXAM SESSION: 9-11.00 AM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions in SECTION B**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE COMPULSORY (30 MARKS)

a) Briefly explain the difference between Bayesian and Frequentist methods (8mks)

b) Let X be a continuous random variable with the following PDF

$$f_x(x) = \begin{cases} 2x & o \le x \le 1 \\ 0 & otherwise \end{cases}$$

Suppose that we know

$$Y/X = x \sim \text{Geometric } (x)$$

Find the MAP estimate of X given Y = 3 (5mks)

c) The value of the posterior distribution is $\pi_{\Theta|X}(\theta | 6) = 0.53$ for all x = 6. For x = 6, the value of the joint distribution $f_{X,\Theta}(x, \theta)$ is 0.045. Find the value of the marginal distribution, given that it is constant for all x. (5mks)

d) The value of the marginal distribution is $f_X(x) = 0.034$ for all x. For x = 5, the value of the joint distribution $f_{X,\Theta}(x, \theta)$ is 0.0019. Find the value of the posterior distribution $\pi_{\Theta|X}(\theta | x)$ at x = 5 (5mks)

e) Suppose that there are two websites, A and B, for renting books. The site A receives 60% of all orders. Among the orders placed on site A, 75% arrive on time. Among the orders placed on site B, 90% arrive on time. Given that an order arrived on time, find the probability that the order was placed on site B. (7mks)

QUESTION TWO (20 MARKS)

a) In a certain small town there are n taxis which are clearly numbered 1, 2, . . . , n. Before we visit the town we do not know the value of n but our probabilities for the possible values of n are as follows.

n	0	1	2	3	4	5	6	7	8	≥ 9
Probability	0.00	0.11	0.12	0.13	0.14	0.14	0.13	0.12	0.11	0.00

On a visit to the town we take a taxi which we assume would be equally likely to be any of taxis 1, 2, . . . , n. It is taxi number 5. Find our new probabilities for the value of n. (10mks)

b) In a small survey, a random sample of 50 people from a large population is selected. Each person is asked a question to which the answer is either "Yes" or "No." Let the proportion in the population who would answer "Yes" be θ. Our prior distribution for θ is a beta(1.5, 1.5) distribution. In the survey, 37 people answer "Yes."

(i) Find the prior mean and prior standard deviation of θ.
(ii) Find the prior probability that $\theta < 0.6$.
(iii) Find the likelihood.
(iv) Find the posterior distribution of θ.
(v) Find the posterior mean and posterior standard deviation of θ.
(vii) Find the posterior probability that $\theta < 0.6$ (10mks)

QUESTION THREE (20 MARKS)

a) A biologist is interested in the proportion, θ, of badgers in a particular area which carry the infection responsible for bovine tuberculosis. The biologist's prior distribution for θ is a beta(1, 19) distribution.

i. Find the biologist's prior mean and prior standard deviation for θ.
ii. Find the cumulative distribution function of the biologist's prior distribution and hence find values $\theta 1$, $\theta 2$ such that, in the biologist's prior distribution, $\Pr(\theta < \theta 1) = \Pr(\theta > \theta 2) = 0.05$.

b) The biologist captures twenty badgers and tests them for the infection. Assume that, given θ, the number, X, of these carrying the infection has a binomial(20, θ) distribution. The observed number carrying the infection is $x = 2$.

i. Find the likelihood function.
ii. Find the biologist's posterior distribution for θ.
iii. Find the biologist's posterior mean and posterior standard deviation for θ.

QUESTION FOUR (20 MARKS)

a) A factory produces large numbers of packets of nuts. As part of the quality control process, samples of the packets are taken and weighed to check whether they are underweight. Let the true proportion of packets which are underweight be θ and assume that, given θ, the packets are independent and each has probability θ of being underweight. A beta(1, 9) prior distribution for θ is used.

The procedure consists of selecting packets until either an underweight packet is found, in which case we stop and note the number X of packets examined, or $m = 10$ packets are examined and none is underweight, in which we case we stop and note this fact.

i. Find the posterior distribution for θ when $X = 7$ is observed.

ii. Find the posterior distribution for θ when no underweight packets are found out of $m = 10$. (10mks)

b) Now consider varying the value of m. Use R to find the posterior probability that $\theta < 0.02$ when no underweight packets are found out of
   i. $m = 10$,
   ii. $m = 20$,
   iii. $m = 30$ (10mks)

QUESTION FIVE (20 MARKS)

a) Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a Bernoulli distribution where $P(Y_i = 1) = p$ and $P(Y_i = 0) = 1 - p$ and assume that the prior distribution for p is beta(α,β). Find the posterior distribution for p. (10mks)

b) A particular species of fish makes an annual migration up a river. On a particular day there is a probability of 0.4 that the migration will start. If it does then an observer will have to wait T minutes before seeing a fish, where T has an exponential distribution with mean 20 (i.e. an exponential (0.05) distribution). If the migration has not started, then no fish will be seen.
(i) Find the conditional probability that the migration has not started given that no fish has been seen after one hour.
(ii) How long does the observer have to wait without seeing a fish to be 90% sure that the migration has not started? (10mks)