



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE

ACTUARIAL

2ND YEAR 1ST SEMESTER 2016/2017 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SAC 203

COURSE TITLE: FUNDAMENTALS OF ACTUARIAL MATHEMATICS 11

EXAM VENUE:

STREAM: (BSc. Actuarial)

DATE:

EXAM SESSION:

TIME: 2.00 HOURS

Instructions:

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION 1 [30 Marks]

a). Given ${}_tP_{30} = \frac{7800 - 70t - t^2}{7800}$ for $0 \leq t \leq 100 - x$

i) Find ${}_{10}P_{30}$ and ${}_1P_{30}$ and hence find ${}_9P_{31}$ [4 marks]

ii) Find μ_{30+t} and then μ_{30} and μ_{40} [4 marks]

iii) Calculate ${}_{10}P_{31}$ assuming μ_{40} to be constant between ages 40 and 41 [2 marks]

b). For a 20 year pure endowment of 1 on (x), you are given that:

i) Z is the present value random variable at the issue of the benefit payment

ii) $\text{Var}(Z) = 0.05E(Z)$ and iii) ${}_{20}P_x = 0.65$

Calculate i [5 marks]

c). A life aged exactly 50 buys a 15 year term assurance policy with a sum assured of kshs.100,000 payable at the end of year of death. Level of premiums are payable monthly in advance. Calculating the monthly premium, assuming AM92 ultimate mortality at 6 % interest. Ignore the expenses. [8 marks]

d). i. Given that $a_x = 20$, $a_{x:n} = 18$, and $a_{x+n} = 8$, find the values of ${}_nE_x$ and $a_{x:n}$ [5 marks]

ii. If $A_{x:n} = a$, $A_{x:n} = b$ and $A_{x+n} = c$ find A_x in terms of a, b, c [2 marks]

3) A life office issued a 20 year endowment assurance to a person aged 25. The sum assured is \$10,000 payable immediately of death or on survival for 20 years, Using A1967-70 ultimate 6 % and ignoring expenses, Calculate the standard deviation of the benefit [7 marks]

QUESTION 2 [20 marks]

a) You are given the following:

${}_{15}P_{45} = 0.038$, $P_{45:15} = 0.056$ and $A_{60} = 0.0625$

Calculate $P_{45:15}$ and interpret this value [6 marks]

b). A life office issued a certain policy to a life aged 40. The benefits under this contract are as follows:

On death before age 60 :an immediate lump sum of \$ 1,000

On survival to age 60:An annuity of \$ 500 p.a payable continuously for the remaining lifetime of the policy holder.

Level annual premiums are payable continuously until age 60 or earlier death. Premiums are calculated according to the following basis:

Mortality :AM92

Interest:4% pa ;Expenses:Nil

Calculate the annual premium [6 marks]

c).An office issues a large number of 25 year endowment assurance to lives aged exactly 40.Level annual premiums are payable throughout the term and the sum assured of each policy is \$ 10,000 payable at the end of year of death or on survival to the end of the term.The office premium basis is:

Mortality:AM92 ultimate

Rate:4%

Expenses:5% of each annual premium including the first with additional expenses of 1% of the sum assured [6 marks]

d).State the conditions under which the prospective and the retrospective reserves will be equal [2 marks]

QUESTION THREE [20 MARKS]

a)Ten years ago a life office issued a 30 year endowment assurance without profits to a person aged 35.The sum assured is \$10,000 payable at the end of year of death or on survival for 30 years, and premiums are payable annually in advance. Using AM92 ultimate 6 % and ignoring expenses, Calculate

- i. The annual premium [4 marks]
- ii. The reserve ,assuming that premium now due has been paid [3 marks]

b.You are given the following values:

$$A_x = 0.2340 , A_{x+40} = 0.5297 , A_{\overline{1}|x:40} = 0.2222 \text{ and } i=0.07$$

Find $\ddot{a}_{x:40}$ [4 marks]

c. Calculate by means of commutation functions on the basis of AM92 tables at 4% the value of :

i. ${}_{10/5}A_{\overline{40}|}$ ii. ${}_{5/}a_{\overline{45}|}$ [2 marks]

5. A life aged exactly 60 wishes to arrange for payment to be made to charity in 10 years time. If he is still alive at that date, the payment will be \$ 10,000. If he dies before the payment date, the amount given will be \$ 500. Assuming an effective rate of 6% per annum and mortality according to AM92 select males. Calculate the standard deviation of the present value of the liability [7 marks]

QUESTION FOUR [20 MARKS]

a). Given that $m_x = \frac{l_x - l_{x+1}}{\frac{1}{2}(l_x + l_{x+1})}$ in the usual life table notation, determine m_x in terms of q_x and

hence show that $p_x = \frac{2 - m_x}{2 + m_x}$ [5 marks]

b). A mortality table is defined such that

$${}_t p_x = \left(1 - \frac{t}{110 - x}\right)^{1/2} \quad \text{for } x < 110, t < 110 - x$$

And ${}_t p_x = 0$, for $t \geq 110 - x$

Calculate:

- i. The complete expectation of life at exact age 45 [4 marks]
- ii. The force of mortality at age 45 [4 marks]

3) In a certain population, ${}_t\mu_x$ is given by

Interval	μ_x
$60 \leq x \leq 70$	0.01
$70 \leq x \leq 80$	0.015
$x > 80$	0.025

Calculate the probability that a life aged exactly 65 will die between ages 80 and 83 [3 marks]

4.) Suppose that $s(x) = (1 - x/\omega)^\alpha$

Give simple formulae for

i. μ_x [2 mark]

ii. e_x^0 [2 marks]

QUESTION FIVE [20 MARKS] a

a.) Suppose $A_x = 0.25$, $A_{x+n} = 0.40$ and $A_{x:20} = 0.55$. Compute $A_{x:20}^1$ and $A_{x:20}^1$ [4 marks]

b.) Using English life table no.15 –male calculate

i. $e_{4:10}$ [2 marks]

ii. T_{20} [2 marks]

4. A life insurance company for many years has recruited, uniformly over each year, 100 staff members on their 21st birthdays and a fixed number of additional employees on their 25th birthdays. Mortality follows English Life table no.-15 males. Employees may retire on their 60th or 65th birthdays, and one third of employees reaching their 60th birthdays retire on that date. Employees leave the company only through death or retirement, and the total number of employees is 10,000. Find the total number of new recruits each year [6 marks]

5. A large manufacturing company has for many years staffed one of its divisions by the recruitment, uniformly over each year, of 1,000 staff at exact age 20. At the end of one year in the job, new staff are examined for suitability, and 20% are dismissed. All employees are assessed at age 35, and 50% are immediately moved out of the division. At age 40, all the remaining employees are moved out of the division. Death is the only other reason for leaving the division. Staff experience mortality according to English Life table No.15 –males. Calculate the number of staff in the division [6 marks]