JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

3RD YEAR 2ND SEMESTER 2016/2017 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SAS 310

COURSE TITLE: STOCHASTIC AND DECISION MODELLING I

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) Define stochastic modeling? (2 Marks)
b) Outline steps involved in stochastic modeling (4 Marks)
c) Explain the importance's of performance measures in a queueing system behavior (8 Marks)
d) Name three techniques for simulating continuous random variables. (3 Marks)
e) Use rejection method to generate a random variable that has density function

$$f(x) = 20x(1-x)^3 \qquad 0 < x < 1$$

With $g(x) = 1 \qquad 0 < x < 1$ (6 Marks)

f) Taxis are waiting in a queue for passengers to come. Passengers for these taxis arrive according to a Poisson process with an average of 60 passengers per hour. A taxi departs as soon as two passengers have been collected or 3 minutes have expired since the first passenger has got a taxi. Suppose you get in the taxi as first passenger. What is your average waiting time for the departure?
g) Outline reasons why we use simulation in any stochastic system. (2marks)

QUESTION TWO (20 MARKS)

Show that W is smaller in a $M/M/1$ model having arrivals at rate λ and service at rate 2μ than it is a two-server $M/M/2$ model with arrivals at rate λ and with each server at a rate μ. Give an intuitive explanation for this result? Would it also be true for W_Q?

QUESTION THREE (20 MARKS)

a) Explain three classes of a Queueing system. (6 Marks)
b) A supermarket has two exponential check out counters, each operating at a rate μ. Arrivals are Poisson at a rate λ; the counters operate in the following ways.
   - One queue feeds both counters.
   - One counter is operated by a permanent checker and the other by a stock clerk who instantaneously begin checking whenever there are two or more customers in the system. The clerk returns the stocking whenever he completes a service, and there are fewer than two customers in the system,

i. Find P_n, proportion of time there are n in the system
ii. At what rate does the number in the system go from 0 to 1? From 2 to 1?
iii. What proportion of time is the stock clerk checking? (9 Marks)

c) The bus that takes you home from Kisumu arrives at the nearest bus station from early morning till late in the evening according to a renewal process with inter arrival times that are uniformly distributed between 5 and 10 minutes. You arrive at the bus station at 5 p.m. Estimate your waiting time for the bus to arrive (5 Marks)

QUESTION FOUR (20 MARKS)

a) Show that if $X_1, X_2, \ldots$ are independent and identically distributed random variables having finite expectations, and if N is a stopping time for $X_1, X_2, \ldots\ldots$ such that $E[N] < \infty$, then $E\left[\sum_{1}^{N} X_n\right] = E[N]E[X]$ (10 Marks)

b) For a non homogeneous Poisson process with intensity functions $\lambda(t), t \geq 0,$ where $\int_0^\infty \lambda(t)dt = \infty,$

let $X_1, X_2, \ldots$ denote the sequence of times at which events occur.

i. Show that $\int_0^{X_1} \lambda(t)dt$ is exponential with rate 1.

ii. Show that $\int_{X_{i-1}}^{X_i} \lambda(t)dt, i \geq 1$, are independent exponentials with rate 1, where $X_0 = 0$

(10 Marks)

QUESTION FIVE (20 MARKS)

Show that if $X_1, \ldots\ldots X_n$ are independent, then, for any increasing functions f and g of n variables, $E[f(X)g(X)] \geq E[f(X)]E[g(X)]$ where $X = (X_1, \ldots\ldots, X_n)$.