JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

2016/2017 ACADEMIC YEAR

YEAR ONE SEMESTER TWO

MAIN REGULAR

APRIL 2017 EXAMINATION

COURSE CODE: SAS 102

COURSE TITLE: PROBABILITY AND DISTRIBUTION THEORY 1

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: EXAM SESSION:

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY)-(30 MARKS)

a) The joint density function of two continuous random variables X and Y is given by

$$f(x,y)=\begin{cases}k(2x-y), & 0\le x\le 2,0\le y\le 3\\ 0, & otherwise\end{cases}$$

Obtain

i. the value of k.

ii. the expected value of X (6marks)

b) Let $f(x,y)=\begin{cases}6x^2y, & 0\le x\le 1,0\le y\le 1\\ 0, & otherwise\end{cases}$

be the p.d.f. of two random variables X and Y, which must be of continuous type. Find $P(0<x<{}^{3}/_{4}, y>{}^{1}/_{3})$ (6marks)

c) The joint probability function for two discrete random variables X and Y is tabulated as shown

	Y=0	Y=1	Y=2	Y=3
X=1	0.06	0.02	0.04	0.08
X=2	0.15	0.05	0.10	0.20
X=3	0.09	0.03	0.06	0.12

Determine

i. Marginal distributions of X and Y . (2marks)

ii. P(X≤2, Y≥2) (2marks)

d) The failure of a circuit board interrupts work until a new board is delivered. The delivery time Y is uniformly distributed on the interval one to five days. The cost of a board failure and interruption C includes a fixed cost C_0 and increases proportionally to the cube of the delivery time Y^3. This cost is modeled by $C=C_0+C_1Y^3$. Find

i. The probability that the delivery time does not exceed 4 days but must take at least one day.

ii. In terms of C_0 and C_1, the expected cost associated with a single failed circuit board. (7marks)

e) Suppose X is a continuous random variable with pdf $f(x)=\begin{cases}5x^4, & 0<x<1\\ 0, & otherwise\end{cases}$

Determine

i. The pdf of the continuous random variable Y where $Y=X^3$

ii. $p(0.5<Y<1)$ (7marks)

QUESTION TWO (20 MARKS)

a) Given $f(x,y)=\begin{cases}2e^{-x-2y}, & 0<x<\infty,0<y<\infty\\ 0, & otherwise\end{cases}$.

Determine

i. $P(X>1,Y<1)$

ii. $P(X<Y=10)$ (9marks)

b) A random variable X has the Beta distribution with parameters α and β as shown below.

$$f(x) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma\alpha\Gamma\beta} x^{\alpha-1}(1-x)^{\beta-1}, & 0 < x < 1, \alpha > 0, \beta > 0 \\ 0, & otherwise \end{cases}$$

Determine by derivation for this distribution, the standard deviation when $\alpha = 8, \beta = 10$. (11marks)

QUESTION THREE (20 MARKS)

a) The joint probability function of two discrete random variables X and Y is given by

$$f(x,y) = \begin{cases} k(2x+y), & 0 \leq x \leq 3, 1 \leq y \leq 3 \\ 0, & otherwise \end{cases}$$

i. Obtain the value of k.
ii. Obtain $E(Y^2)$
iii. Deduce whether or not X and Y independent? (10marks)

b) Consider the Weibull distribution with parameters a and b

$$f(x) = \begin{cases} abx^{b-1}e^{-ax^b}, x > 0 \\ 0, \ otherwise \end{cases}$$

Obtain a general expression for the mean and the third raw moment for the distribution. (10marks)

QUESTION FOUR (20 MARKS)

a) The joint p.d.f of three continuous random variables X , Y and Z is defined as follows

$$f(x,y,z) = \begin{cases} k(xy+z), & 0 < x < 3, 0 < y < 4, 0 < z < 1 \\ 0, & otherwise \end{cases}$$

Calculate:
i. the value of k ,
ii. the marginal distribution of X
iii. $E(YZ/X = 2)$ (14marks)

b) Determine the value of c for which the function below is a joint probability density function.

$$f(x,y) = \begin{cases} c(x+y), & 0 < x < 3, x < y < 2x+1 \\ 0, & otherwise \end{cases}$$

(6marks)

QUESTION FIVE (20 MARKS)

a) A random variable Y has a probability density function given by $f(y) = \begin{cases} cy^3e^{-y/2}, & y > 0, \\ 0, & otherwise \end{cases}$.

Find C hence show that Y has a chi-square distribution. State the degrees of freedom. (10 marks)

b) Let X and Y be two independent standard normal random variables. Suppose $U = X + Y$ and $V = 2X - Y$ are two new random variables in terms of X and Y. Determine the joint pdf of U and V. (10 marks)