



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

**UNIVERSITY EXAMINATION FOR DEGREE OF MASTER OF SCIENCE IN PURE
MATHEMATICS**

1st YEAR 2nd SEMESTER 2016/2017 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SMA 812

COURSE TITLE: ABSTRACT INTEGRATION II

EXAM VENUE: STREAM: (Msc. Pure Mathematics)

DATE: EXAM SESSION: TWO

TIME: 3.00 HOURS

Instructions:

- 1. Answer any THREE questions only**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE [20 MARKS]

- a) State and prove Caratheodory's Extension Theorem. (8 marks)
- b) State and prove the Lebesgue's Decomposition Theorem. (8 marks)
- c) Describe product measure spaces giving relevant examples. (4 marks)

QUESTION TWO [20 MARKS]

- a) Define: L-measurable function, Pointwise convergence and Cauchy sequence. (4 marks)
- b) State Holders inequality and Minkowski's inequality. (4 marks)
- c) Describe convergence in measure. (4 marks)
- d) State and prove Radon-Nikodym Theorem. (8 marks)

QUESTION THREE [20 MARKS]

- (a) State and prove Caratheodory's Extension Theorem. (7 marks)
- (b) State and prove the Lebesgue's Decomposition Theorem. (7 marks)
- (c) Prove that strong convergence implies weak convergence in measure spaces. (6 marks)

QUESTION FOUR [20 MARKS]

- a) Are Borel spaces measure spaces? Explain your assertion. (10 marks)
- b) State and prove Lebesgue's Dominated Convergence Theorem for complex spaces and real spaces. (10 marks)
- c)

QUESTION FIVE [20 MARKS]

- a) By giving relevant application areas, state and prove Tonelli's Theorem. (8 marks)
- b) State and prove Fubini's Theorem. (6 marks)
- c) State and prove Spectral Mapping Theorem for measure spaces. (6 marks)