JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR BED AND ACTUARIAL SCIENCES
2nd YEAR 2nd SEMESTER 2016/2017 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SMA201
COURSE TITLE: LINEAR ALGEBRA II

EXAM VENUE: **STREAM: BED AND ACT SCIENCE Y2S2**

TIME: 2 HOURS **EXAM SESSION:**

Instructions:

1. **Answer question1 and any other 2 questions.**
2. **Candidates are advised not to write on the question paper**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

Question1 [30marks] Compulsory

(a) (i)Given matrix $M = \begin{bmatrix} 11 & 0 & 12 \\ 6 & 24 & 0 \\ 1 & 0 & 10 \end{bmatrix}$

Compute M^t the transpose of M and determinant of M. [5marks]

(ii)If $A = \begin{bmatrix} 11-4i & i & 12 \\ 6+2i & 24+i & 0 \\ 1 & i & i \end{bmatrix}$

Find the matrix A^* the ***adjoint*** of **A**. [5marks]

(b) Suppose the mapping $L: R^2 \to R^2$ with $L\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x-y \\ x+y \end{bmatrix}$

(i) Show that L is linear. [6marks]

(ii)Determine A the matrix of L with respect to the ordered basis $\left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right\}$ [7 marks]

(c) Let the binary rules $\oplus, \otimes$ be defined on the R^2 vector space by : $\begin{pmatrix} x \\ y \end{pmatrix} \oplus \begin{pmatrix} u \\ v \end{pmatrix} = xu + yv$

; $\begin{pmatrix} x \\ y \end{pmatrix} \otimes \begin{pmatrix} u \\ v \end{pmatrix} = xu - yv - 5$

$(i) Compute$ $\begin{pmatrix} 0 \\ 0 \end{pmatrix} \oplus \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} x \\ y \end{pmatrix} \oplus \begin{pmatrix} x \\ y \end{pmatrix}$, $\begin{pmatrix} x \\ y \end{pmatrix} \otimes \begin{pmatrix} x \\ y \end{pmatrix}$: $\begin{pmatrix} x \\ y \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ **[7marks]**

(ii)State giving reasons which of the rules $\oplus, \otimes$ is not an inner product on the R^2 vector space[7marks]

Question2 [20marks]

(a)(i) Without using direct computation , show that $-3, -3, 1$ eigenvalues of

the matrix $A=\begin{pmatrix} 1 & -4 & -4 \\ 8 & -11 & -8 \\ -8 & 8 & 5 \end{pmatrix}$. [4marks]

(ii) Verify that $[A+3I]^2[A-I]=\underline{0}_{3\times 3}$ [5marks]

(b) (i) For what values of the constants a,b,d

does the matrix equation $a\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -4 \\ 0 & 4 & 0 \end{bmatrix}+b\begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 0 \\ -4 & 0 & 0 \end{bmatrix}+d\begin{bmatrix} 0 & -4 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ hold ? [4marks]

(ii) Determine the specific values of the constants a,b,d

such that the set of 3by3 matrices $\left\{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -4 \\ 0 & 4 & 0 \end{bmatrix},\begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 0 \\ -4 & 0 & 0 \end{bmatrix},\begin{bmatrix} 0 & -4 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$ is linearly independent

[7marks]

Question3 [20marks]

(a) Let f be the form on $V\times V$ such that V is a real vector space. Define $A=(a_{ij})$, the matrix of f w.r.t an ordered basis $\beta=\{\beta_1,\beta_2,,,\beta_n\}$ by $A=(a_{ij})=f(\beta_i,\beta_j)\,i,j=1,2,,,n$. [8marks]

(b)Suppose f is a form on R^2 defined by

$f((x_1,x_2),(y_1,y_2))=x_1y_1+4x_2y_2+2x_1y_2+2x_2y_1$.

Find the matrix of f in each of the bases

(i) $\{[1,-1],[1,1]\}$ **(ii)** $\{[1,0],[0,1]\}$ [1**2 marks]**

Question4 [20 marks]

Consider the vector space of R^4 with the inner product $\langle\ ,\ \rangle$:

$\langle \underline{x}, \underline{y} \rangle = 4x_1y_1 + 4x_2y_2 + x_3y_3 + x_4y_4$; $\underline{x} = [x_1, x_2, x_3, x_4]$, $\underline{y} = [y_1, y_2, y_3, y_4]$, $\underline{0} = [0,0,0,0]$,
$x_i, y_i \in R$; $\underline{x}$, $\underline{y} \in R^4$

(a) Show that $\langle \underline{x}, \underline{x} \rangle > 0$ **[4 marks]**

(b) Show that $\langle \underline{x}, \underline{y} \rangle = \langle \underline{y}, \underline{x} \rangle$ **[4 marks]**

(c) Determine $\langle \underline{x}, \underline{0} \rangle$, $\langle \underline{0}, \underline{y} \rangle$, $\langle \underline{0}, \underline{0} \rangle$ **[4 marks]**

(d) Apply the Gram-Schmidt process to the set of linearly independent vectors

$\{v_1 = [1,1,-1,-1], v_2 = [1,1,1,1], v_3 = [-1,-1,-1,1], v_4 = [1,0,0,1]\}$
to obtain orthogonal set of vectors $\{w_1, w_2, w_3, w_4\}$. **[8 marks]**

Question5 [20marks]

Let B be the matrix of linear operator T on n -dimensional vector space V over F with respect to the standard ordered basis for V .

(a) Explain what is meant by (i) v is an eigenvector of T , $v \in V$ (ii) λ is an eigenvalue of T , $\lambda \in F$ [6 marks]

(b) State the relationship between T and B [2marks]

(c) If matrix $B = \begin{pmatrix} 1 & -9 \\ -1 & 1 \end{pmatrix}$ find

(i) λ_1 , λ_2 the eigenvalues of T and v_1 , v_2 the corresponding eigenvectors [8marks]

(d) Confirm that B is diagonalizable. [2marks]

(e) Diagonalize matrix B . [2marks]

SMA 201: Linear Algebra II (42hrs)

Credit Hours: 3

Pre-requisites Linear Algebra I

Purpose

To apply basic algebra concepts in understand Matrix analyses and Eigen value theorems and to extend linear algebra I to vector spaces and concepts of diagonalization and decomposition

Expected Learning Outcomes

By the end of the course the learner should be able to:

i) Evaluate determinant
ii) Represent a function by matrix
iii) Use Eigenvalue and Eigenvectors to determine whether a given matrix is diagonalizable
iv) Orthogonalize sets of vectors

Course Content

Field axioms. Vector spaces over an arbitrary field. Linear mapping and their matrices with respect to an arbitrary basis, the change of basis. Conjugation of eigenvectors theorem. Invariant subspaces. Quadratic forms.

Inner product spaces

.**Teaching / Learning Methodologies:** Lectures; Practical work (Minitab Package); Class discussion

Instructional Materials and Equipment: Overhead Projectors; Power Point; Flip Charts; Handouts; Chalk board

Course Assessment: Examination - 70%; Continuous Assessments (Exercises and Tests) - 30%; Total - 100%

Core reading materials

i) Howard Anton (2008); *Elementary Linear Algebra*; John Wiley
ii) Stephen H Friedberg, Arnold J Insel, Lawrence E Spence (2004); *Linear Algebra*; Prentice-hall Of India Pvt Ltd
iii) Klaus Janich (2004); *Linear Algebra*; Springer Verlag
iv) Lipschut, Seymour (1991); *Theory and Probability;* Scham Series McGraw Hill New York
v) Semesi E (1993); *Linear Algebra, a Geometry Approach;* Chapman and Hall London.

Recommended reference materials

i) David C. (2004); *Linear Algebra and its Applications*; Addition Wesley Publishers
ii) Towers, David (1988); *Guide to linear Algebra;* McMillan Press London
iii) Morris A.O. (1978); *Linear Algebra an Introduction;* Chapman and Hall London.