JARAMOGI OGINGA ODINGA UNIVERSITY OF
SCIENCE AND TECHNOLOGY

UNIVERSITY ***DRAFT*** **RESIT/RETAKE**
EXAMS 2015/2016

SCHOOL OF MATHEMATICS ,APPLIED STAT. AND ACTUARIAL SCIENCES
SEMESTER ONE, SECOND YEAR EXAMINATIONS

SEMESTER 11 SECOND YEAR BSC, BEd EXAMS

SEMESTER TWO, THIRD YEAR EXAMINATIONS

FOR THE DEGREE

OF

BACHELOR OF EDUCATION/ SCIENCE

COURSE CODE: SMA 304

COURSE TITLE: GROUP THEORY

DATE: May, 2016 TIME: 2hrs

INSTRUCTIONS

Attempt **Question1** and **two other** questions

Show all the necessary working

Question1 [30 marks] COMPULSORY

(a) Define a relation $\square$ on Z, the set of integers, by $m \square n$ if $m-n$ is divisible by 5.
Show that
(i) $\square$ is reflexive (ii) $\square$ is symmetric (iii) $\square$ is transitive
(iv) $\square$ defines an equivalence relation on Z [4 marks]

(b) If $a \equiv b \bmod 7$, $c \equiv d \bmod 7$ and $(m,7)=1$ prove that
(i) $a+c \equiv b+d \bmod 7$ (ii) $a-c \equiv b-d \bmod 7$ (iii) $ma \equiv mb \bmod 7$ [10 marks]

(c) Consider the multiplication in $Z/7$ as shown below.

$\times$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2					
3	3		2			
4	4					
5	5				4	
6	6					1

(i) Complete the multiplication table
(ii) Show that $(5\times 4)\times 6 = 5\times(4\times 6)$
(iii) State the neutral element
(iv) Determine the inverse of each element. [10marks]

(d) Let $\langle G,\oplus\rangle$, $\langle \bar{G},*\rangle$ be groups defined on the sets G, $\bar{G}$ respectively.
If $\theta: G \to \bar{G}$ is a mapping defined from G into $\bar{G}$, state what is meant by
(i) θ is a homomorphism
(ii) θ is an isomorphism
(iii) K_θ is kernel of θ [6 marks]

Question 2 [20 marks]

Define the product $x \oplus y$ by , $x+y$ on the set R of real numbers and the product $x * y$ by $x \times y$ on the set R^* of nonzero real numbers .

(a) Prove that $\langle R, \oplus \rangle$,$\langle R^*, * \rangle$ are groups . [10 marks]

(b) Suppose $\Phi : R \to R^*$ *with* $\Phi(r) = 3^r;\ r \in R$.

(i) show that Φ is a homomorphism

(ii) show that Φ is a one-to-one

(iii) determine K_Φ the kernel of Φ [10 marks]

Question3 [20 marks]

Let $\langle G, \bullet \rangle$, $\langle H, * \rangle$ be finite groups defined on the sets G , H with binary operations $\bullet$, $*$ respectively.

If $\theta : G \to H$ is a homomorphism mapping defined from G into H, prove that K_θ the kernel of θ, is a normal subgroup of G. [20 marks]

Question 4 [20 marks]

A mapping p is defined on a set of permutations by $p:(1234) \to (2341)$. Determine the least positive integer m such that $(p)^m:(1234) \to (1234); 1 \le m \le 24$. Let $p_j = (p)^j : j = 1, 2, 3, \ldots, m$ and take $p_1 \bullet p_2$ to mean p_1 followed by p_2 ; $p_1 \bullet p_1 = (p_1)^2$ to mean p_1 followed by p_1; $p_1 \bullet p_2 \bullet p_3$ to mean p_1 followed by p_2 followed by p_3 etc .

Construct Cayley table for the structure $\langle S, \bullet \rangle$ where $S = \{p_1, p_2, p_3, \ldots, p_m\}$.

(i) Show that $\bullet$ is closed on the set S

(ii) Show that $(p_1 \bullet p_2) \bullet p_3 = p_1 \bullet (p_2 \bullet p_3)$

(iii) State the neutral element p_k

(iv) Determine the inverse of each element p_j . [20 marks]

Question 5 [20 marks]

Let $\langle G, \bullet \rangle$,$\langle H, * \rangle$ be groups defined on the sets G , H with binary operations $\bullet$, $*$ respectively.

Define $G \times H$ by the ordered pair (g_1, h_1) for which $G \times H = (g_1, h_1) : g_1 \in G, h_1 \in H$.

If $(g_1, h_1) \otimes (g_2, h_2) = (g_1 \bullet g_2,\ h_1 * h_2) : g_1, g_2 \in G, h_1, h_2 \in H$ with $\otimes$, as a new binary operation on $G \times H$, prove that the direct product $\langle G \times H, \otimes \rangle$ forms a group. [20 marks]