JARAMOGI OGINGA ODINGA UNIVERSITY OF
SCIENCE AND TECHNOLOGY

UNIVERSITY ***DRAFT*** **RESIT/RETAKE**
EXAMS 2015/2016

SEMESTER TWO FOURTH YEAR Bed/BSc

SMA414: FOURIER ANALYSIS

May , 2016 **Time: 2hrs**

INSTRUCTIONS

Answer **Question1** and **two** other questions
Show all the necessary working

Question.1 [30 marks] Compulsory

(a) Assume that $f(x)$ has a uniformly convergent Fourier series

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \{a_n \cos(nx) + b_n \sin(nx)\} \quad -L \le x \le L$$

Describe what is meant by Gibb's phenomenon of the periodic function $f(x)$. [9 marks]

(b) Given the function $f(x) = \begin{cases} x & -15 < x < 15 \\ f(x+30) & otherwise \end{cases}$

(i) Sketch graph of $f(x)$ over the interval $-60 < x < 60$

(ii) State period of $f(x)$ [11 marks]

(c) Obtain the Fourier **half-range** sine coefficients for this function $f(x) = 1, 0 < x < \pi$.

Sketch the anti-symmetric odd periodic extension of f(x) on (- π , π) [10 marks]

Question2 [20 marks]

Find the general solution to the first order ordinary differential equation

$$y' + y = \frac{1}{2}e^{-|t|}, \quad -\infty < t < \infty$$ **[20 marks]**

Question3 [20 marks]

Given real valued function $f(x)$ for which

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

$$f(x) = f(x+2\pi)$$

(a)Sketch the graph of $f(x)$ over the interval $-14\pi < x < 14\pi$ [5 marks]

(b)State period of $f(x)$ [3 marks]

(c)Deduce that

$$f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left\{\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \frac{1}{49}\cos 7x...\right\} + \sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \frac{1}{4}\sin 4x...$$

[12 marks]

Question4 [20 marks]

(a) Obtain the Fourier series for the function

$$f(x) = \begin{cases} \pi & ; \quad 0 \le x < \pi \\ -\pi & ; \quad -\pi \le x < 0 \end{cases}$$

(b) Show that the infinite series for $F(x) = \int_{-\pi}^{x} f(t)dt$ converges uniformly .

(c) Show that the infinite series for $g(x) = \dfrac{df(x)}{dx}$ does not converge.

Question5 [20 marks]

(a) State and prove the Parseval's Theorem

(b) Consider the function defined on [0, π) by

$$f(x)=\begin{cases}\pi - x & 0 \le x < \dfrac{\pi}{2} \\ x & \dfrac{\pi}{2} \le x < \pi\end{cases}$$

Obtain the Fourier cosine coefficients for this function $f(x)$.

Sketch the symmetric even periodic extension of f(x) on (-π , π)