JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SPECIAL RESIT EXAMINATION-APRIL/MAY 2016

SMA 403: TOPOLOGY

INSTRUCTION: Attempt question one (**COMPULSORY**) and any other TWO questions only.

QUESTION ONE(*COMPULSORY*) [30 MARKS]

(a). Show that the Sierpinski's space is a T_0-space. (3 marks)

(b). Show that $f : [0, 1] \to [0, 5]$ defined by $f(x) = 5x$ is a homeomorphism. (5 marks)

(c). Distinguish between a regular space and a base. (4 marks)

(d). Show the continuity of the constant function . (4 marks)

(e). Show that a set is open if its complement is closed. (4 marks)

(f). Let X be a topological space, and let A be a subset of X. A subset B of A is closed in A if and only if $B = A \setminus F$ for some closed subset F of X. (5 marks)

(g). Define: Neighbourhood, closed set and limit point. (5 marks)

QUESTION TWO [20 MARKS]

(a). Show that the set $C_R[0, 1]$ of all real-valued continuous functions defined on $[0, 1]$ is a metric space with respect to the metric defined as $d(f, g) = \max\{|f(x) - g(x)|; x \in [0, 1]\}$ where $f, g \in C_R[0, 1]$. (10 marks)

(b). Show that $(X, \mathfrak{D})$ is a topological space where $\mathfrak{D}$ is the class of all subsets of X. (10 marks)

QUESTION THREE [20 MARKS]

(a). State and prove the cardinality theorem for any three finite sets A, B and C. (5 marks)

(b). Prove that all metric spaces are Hausdorff spaces. (15 marks)

QUESTION FOUR [20 MARKS]

(a). Give the condition for normality of a space. (2 marks)

(b). If $P = \{(a, b) \in \mathbb{R}^2 : ab = 1\}$ and $Q = \{(a, b \in \mathbb{R}^2 : b = 0)\}$, find $d(P, Q)$. (5 marks)

(c). State and prove Cantor's Cardinality Theorem. (9 marks)

(d). Describe two practical applications of the study of topology. (4 marks)

QUESTION FIVE [20 MARKS]

(a). Define continuity of a function between topological spaces (2 marks)

(b). Let X, Y, Z be topological spaces, and let $f : X \to Y$ and $g : Y \to Z$ be continuous functions. Then the composition $g \circ f : X \to Z$ of the functions f and g is continuous. (14 marks)

(c). Let X, Y be topological spaces, and let $f : X \to Y$ be a function from X to Y. The function f is continuous if and only if $f^{-1}(G)$ is closed in X for every closed subset G of Y. (4 marks)