JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

SPECIAL RESIT 2015/2016 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SMA 301

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATION I

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY)

a) Given $y = A\sin x + B\cos x$, where A and B and arbitrary constants, eliminate the arbitrary constants to form a differential equation hence state its order and degree. (5 marks)

b) The rate of cooling of a body is proportional to the excess of its temperature above its surrounding θ^0 C. A body cools from 85^0C to 65^0C in 4.0 minutes at a surrounding temperature of 15^0C. Determine how long to the nearest second the body will take to cool to 55^0C. (4 marks)

c) Solve the differential equation below using an appropriate method

$$\frac{d^2y}{dx^2} + 36y = 0$$ (5 marks)

d) Using an appropriate method solve the differential equation $2yy'' = 1 + y'$.

(5 marks)

e) Use the method of variation of parameters to solve $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = e^{2x}$.

(5 marks)

f) Solve the differential equation $(y - 2x - 4)dy = (y + 2x - 2)dx$ (6 marks)

QUESTION TWO (20 marks)

a) By finding the integrating factor , find the general solution of the differential equation

$\frac{(1-x^2)}{x}\frac{dy}{dx} + \frac{2x^2 - 1}{x^2}y = x$ (Hint: Use partial fractions) (10 marks)

b) A resistance (R) of 100 ohms, an inductance (L) of 0.5 henry are connected in series with a battery of 20 volts(V). Find the current (i) in the circuit as a function of time(t) given that they are connected by the differential equation $Ri + L\frac{di}{dt} = V$. (5 marks)

c) Solve the differential equation below using any appropriate method

$$4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = e^x.$$ (5 marks)

QUESTION THREE

a) Consider a second order differential equation

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = F(x)$$,

Let F(x) = 0 and let y = U and y = V , where U and V are functions of x be two solutions to the differential equation, then show that y = (U + V) is also a solution.

(6 marks)

b) Find the general solution of the differential equations

(i) $(xy - x^2)dy + y^2dx = 0$ (4 marks)

(ii) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0$ (4 marks)

(iii) $\frac{d^2y}{dx^2} - 36y = 2\cos 4x$ (4 marks)

QUESTION FOUR

Use any appropriate method to solve each of the differential equations below

a) $\left(2-9xy^2\right)xdx+\left(4y^2-6x^3\right)ydy=0$ given that $y = 4$ when $x = 1$. (6 marks)

b) $yy''+(y')^2=0$ (6 marks)

c) $\frac{dy}{dx}+\frac{x}{1-x^2}y=x\sqrt{y}$ (8 marks)

QUESTION FIVE

a) Detectives discovered a murder victim at 6.30 am and the body temperature of the victim was then 26 ^{0}C. After 30 minutes the police surgeon arrived and found the body temperature to be 23 ^{0}C. If the air temperature was 16 ^{0}C throughout and the normal body temperature is 37 ^{0}C. At what time did the police surgeon estimate that the crime occurred. (10 marks)

b) Solve the differential equation $xy''=y'+(y')^3$ given $x=1$ when $y=0$ and $x=2$ when $\frac{dy}{dx}=1$ (10 marks)