



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE
UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE
ACTUARIAL
SPECIAL RESIT 2015/2016 ACADEMIC YEAR
MAIN REGULAR

COURSE CODE: SMA 303

COURSE TITLE: COMPLEX ANALYSIS

EXAM VENUE: **STREAM: (BSc. Actuarial/ Education)**

DATE: 04/05/16 **EXAM SESSION: 2.00 – 4.00 PM**

TIME: 2.00 HOURS

Instructions:

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY) – 30 MARKS

- a) Define each of the following terms as used in complex analysis
- i) Disk
 - ii) Deleted neighbourhood
 - iii) Argument
 - iv) Limits of a complex function (8 marks)
- b) Express $2 - 2\sqrt{3}i$ in polar form using the principal argument. (2 marks)
- c) Evaluate the integral $\oint_C \frac{z}{z^2 + 16} dz$, where C is the circle $|z - 2i| = 4$ using the Cauchy's integral formular. (4 marks)
- d) Compute the n^{th} root for the $(2\sqrt{3} + i)^{\frac{1}{2}}$, hence sketch an appropriate circle indicating the roots w_0 and w_1 . (4 marks)
- e) Sketch the set S denoted by the inequality $2 \leq |z - 3 + i| < 3$. (4 marks)
- f) Find the image of a line $x = 1$ under the complex mapping $w = z^2$ for $w, z \in \mathbb{C}$, hence sketch the line and its image under the mapping (4 marks)
- g) Evaluate the line integral $I = \oint_C (x^2 dx - 2y dy)$ where C comprises the triangle $O(0,0)$, $A(2,1)$ and $C(1,3)$ (4 marks)

QUESTION TWO (20 MARKS)

- a) Prove that if a complex function $f(z) = u(x, y) + iv(x, y)$ is analytic at any point z , and in the domain D , then the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, can be verified. (7 marks)
- b) Find the derivative of $\frac{z^2 - 2iz}{2z + 4 - i}$ (3 marks)

- c) Solve for w , given the complex function $e^w = \sqrt{3} + i$ for $w, \in \mathbf{C}$. (6 marks)
- d) Compute the principal value of the complex logarithm $\ln z$ for $z = 1 - i$ (4 marks)

QUESTION THREE (20 MARKS)

- a) State De-Moivre's theorem hence use it to evaluate $(1 - i)^6$, giving your answer in the form $a + bi$, $a, b \in \mathbf{R}$ (7 marks)
- b) Find an upper bound for the reciprocal of $z^4 - 5z + 1$, given that $|z| = 2$. (5 marks)
- c) Use the definition of the derivative of a complex function to determine the derivative of $f(z) = \frac{1}{z}$ in the region where the derivative exists. (5 marks)
- d) Evaluate $\left(\frac{2+i}{\sqrt{3}+i} \right)^{\frac{1}{4}}$, giving all your answers in polar form. (6 marks)

QUESTION FOUR (20 MARKS)

- a) Find the value of i^i (4 marks)
- b) Given that $e^{i\theta} = \cos \theta + i \sin \theta$ for any real number θ , prove that $e^{iz} = \cos z + i \sin z$ for any complex number z . (6 marks)
- c) Evaluate $\oint_C \frac{1}{z} dz$, where C is the circle $x = \cos t, y = \sin t$ for $0 \leq t \leq 2\pi$ (4 marks)
- d) State the Cauchy's integral formula for derivatives hence evaluate

$$\oint \frac{z^2 + 3}{z(z - i)^2} dz \quad (6 \text{ marks})$$

QUESTION FIVE (20 MARKS)

- a) Find the real numbers p and q for which the complex numbers $z = a + bi$ and $w = a + \frac{1}{b}i$ are equal given that $w, z \in \mathbf{C}$. (3 marks)
- b) Show that the function $f(z) = 3x^2y^2 - 6ix^2y^2$ is not analytic at any point but differentiable along the coordinate axes. (6 marks)
- c) Use L'Hopital's rule to compute

$$\lim_{z \rightarrow 1+i} \frac{z^5 + 4z}{z^2 - 2z + 2} \quad (5 \text{ marks})$$

- d) Given the complex function $f(z) = u(x, y) + iv(x, y)$, verify that the function $u(x, y) = 2x - 2xy$, hence find $v(x, y)$ the harmonic conjugate u , Hence find the corresponding analytic function $f(z) = u + iv$. (6 marks)