JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION 2012/2013

1^{ST} YEAR 1^{ST} SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (SCIENCE) WITH IT

(SCHOOL BASED)

COURSE CODE: SPH 313

TITLE: CLASSICAL MECHANICS

DATE: 3/5/2013 **TIME: 9.00-11.00AM**

DURATION: 2 HOURS

<u>INSTRUCTIONS</u>

1. **Answer ALL questions in Section A**
2. **Answer ANY two Questions from Section B**
3. **Use illustrations where possible**

QUESTION ONE (30 Marks)

a. Show that equation of motion for the kinetic energy for a single particle with constant mass is given by the differential equation

$$\frac{dT}{dt} = F.v$$

While if the mass varies with time then the corresponding equation is given by

$$\frac{d(mT)}{dt} = F.p$$

(4 marks)

b. Prove that the magnitude R of the position vector for the center of mass from an arbitrary origin is given by the equation

(4 marks)

$$M^2R^2 = M\sum_i m_i r_i^2 - \frac{1}{2}\sum_{i,j} m_i m_j r_{ij}^2$$

c. A *central force* is defined to be a force that points radially and whose magnitude depends only on r. That is, $\mathbf{F}(\mathbf{r}) = F(r)\hat{\mathbf{r}}$.

Show that a central force is conservative by explicitly showing that

$\nabla \times \mathbf{F} = \mathbf{0}$. (4 Marks)

d. A bead, under the influence of gravity, slides along a frictionless wire whose height is given by the function $V(x)$, as shown in Fig 1.1.

Find an expression for the bead's horizontal acceleration, $\ddot{x}$

(4 marks)

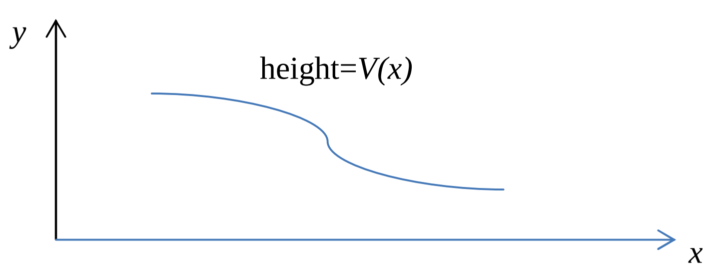


Fig 1.1

e. Consider the pulley system in figure 1.2 with masses $\boldsymbol{M_1}$ **=4kg** and $\boldsymbol{M_2}$**=10kg.** The strings and pulleys are massless. Determine the common acceleration of the masses and the tension in the string? (4 marks)

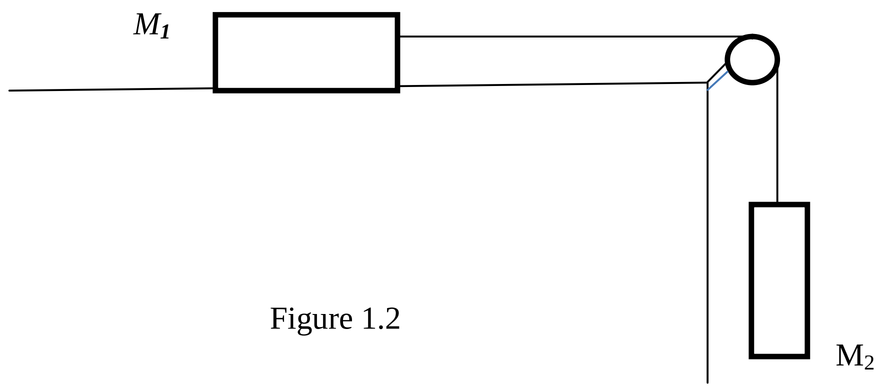


Figure 1.2

f. A billiard ball collides elastically with an identical stationary one. Use the fact that $mv^2/2$ may be written as $m(\mathbf{v} \cdot \mathbf{v})/2$ to show that the angle between the resulting trajectories is 90°. (3 Marks)

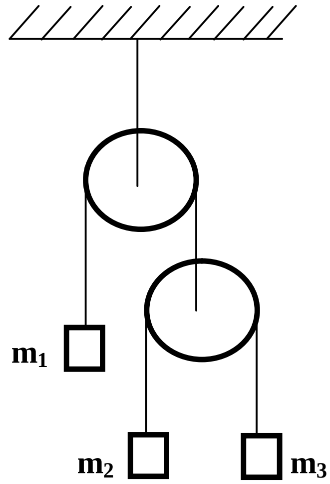
g. The escape velocity of a particle on Earth is the minimum velocity required at Earth's surface in order that that particle can escape from Earth's gravitational field. Neglecting the resistance of the atmosphere, the system is conservative. From the conservation theorem for potential plus kinetic energy show that the escape velocity for Earth, ignoring the presence of the Moon, is 11.2 km/s. (3marks)

h. Briefly explain the concept of time dilation and length contraction with reference to theory of relativity (4 marks)

QUESTION TWO (20 Marks)

a. Mass M_1 is held on a plane with inclination angle θ to the horizontal, and mass M_2 hangs freely vertically over the side. The two masses are connected by a massless string which runs over a massless pulley. The coefficient of kinetic friction between M_1 and the plane is μ. M_2 is released from rest.
Assuming that M_2 is sufficiently large so that M_1 gets pulled up the plane, Determine
i) The common acceleration of the masses
ii) The tension in the string

b. A double Atwood's machine is shown in Figure 2.1, with masses $m1$, $m2$, and $m3$. Find the accelerations of the masses.



QUESTION THREE (20 Marks)

a. Consider a uniform thin disk that rolls without slipping on a horizontal plane. A horizontal force is applied to the center of the disk and in a direction parallel to the plane of the disk.

i. Derive Lagrange's equations and find the generalized force.

ii. Discuss the motion if the force is not applied parallel to the plane of the disk.

(10 marks)

b. A particle of mass m moves in one dimension such that it has the Lagrangian

$$L = \frac{m\dot{x}^4}{12} + mx^2V(x) - V_2(x)$$

Where V is some differentiable function of x.

Find the equation of motion for *x(t)* and describe the physical nature of the system on the basis of this system.

(10 marks)

QUESTION FOUR (20 Marks)

a. Two points of mass m are joined by a rigid weightless rod of length l, the center of which is constrained to move on a circle of radius a. Express the kinetic energy in generalized coordinates (10 marks)

The point of suspension of a simple pendulum of length l and mass m is constrained to move on a parabola $z = ax^2$ in the vertical plane.

Derive a Hamiltonian governing the motion of the pendulum and its point of suspension.

(10 marks)

QUESTION FIVE (20 Marks)

a) Derive the Lorentz's transformations (10 Marks)

b) Briefly discuss the postulates of the theory of relativistic mechanics (10 Marks)