



**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**  
**SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES**  
**UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION**  
**(SCIENCE)**  
**3<sup>RD</sup> YEAR                      1<sup>ST</sup> SEMESTER**  
**MAIN**  
**REGULAR**

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**COURSE CODE: SPH 313**

**COURSE TITLE: CLASSICAL MECHANICS**

**EXAM VENUE:**

**STREAM: (BED SCI)**

**DATE:**

**EXAM SESSION:**

**TIME: 2:00HRS**

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**Instructions:**

- 1. Answer question 1 (Compulsory) in Section A and ANY other 2 questions in Section B.**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

**QUESTION ONE (30 MARKS)**

- a. Three blocks of masses  $m_1= 28\text{kg}$ ,  $m_2 =40 \text{ kg}$  and  $m_3=80\text{kg}$  are connected by two light inelastic strings that passes over a frictionless pulleys, as shown in Figure 1.  $M_2$  is sliding on a rough plane whose coefficient of friction is 0.35.

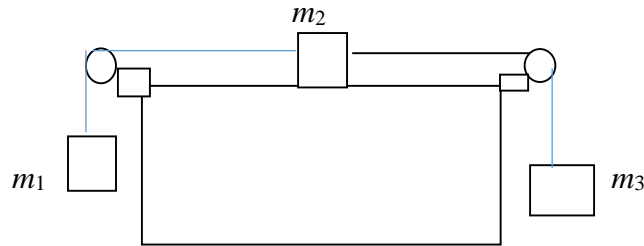


Figure 1

Find the common acceleration of the three blocks and the tensions on the two strings.

**(5 marks)**

- b. Show that the total kinetic energy of a system of many particles is the sum the kinetic energy of the Centre of Mass motion and the kinetic energy relative to the Centre of Mass. (5 marks)
- c. Using the Lagrangian formulation, obtain the equations of motion for a particle of mass  $m$  suspended on a spring pendulum of length  $l$ . (5 marks)
- d. Derive the Hamiltonian equation of a system hence or otherwise obtain the hamiltonian of a free particle moving in one direction  $x$  and described in a uniform frame being accelerated by acceleration  $a$ . **( 5 marks)**
- e. Present the analytical concept of **twin paradox** (6 marks)
- f. Briefly explain the concept of time dilation and length contraction with reference to theory of relativity (4 marks)

**QUESTION TWO (20 Marks)**

- a. Mass  $M_1$  is held on a plane with inclination angle  $\theta$  to the horizontal, and mass  $M_2$  hangs freely vertically over the side. The two masses are connected by a massless string which runs over a massless pulley. The coefficient of kinetic friction between  $M_1$  and the plane is  $\mu$ .  $M_2$  is released from rest. Assuming that  $M_2$  is sufficiently large so that  $M_1$  gets pulled up the plane, Determine
- i) The common acceleration of the masses
  - ii) The tension in the string

**QUESTION THREE (20 Marks)**

- a. The shell theorem states that a uniform shell of matter attracts an external particle as if all the shell's mass were concentrated at its center. Give the mathematical proof of this theorem (8 marks)
- b. A double pendulum consists of two masses  $m_1$  and  $m_2$ . The length of the string supporting  $m_1$  is  $l_1$  while the length of the string from mass  $m_1$  to  $m_2$  is  $l_2$ .  $m_1$  is inclined to the vertical at  $\theta$  while  $m_2$  is inclined at  $\alpha$  to the vertical
- i.) Obtain the Lagrangian of the system (6 marks)
- ii.) Obtain the equations of motion to the system (6 marks)

**QUESTION FOUR (20 Marks)**

- a. Show that the shortest path between two points in a plane is a straight line. (6 marks)
- b. A bead is released from rest at the origin and slides down a frictionless wire that connects a point  $(x,y)$  on the plane to the origin  $(0,0)$ . You wish to shape the wire so that the bead reaches the endpoint in the shortest possible time. Let the desired curve be described by the function  $y(x)$ , with downward taken to be positive.
- i) Show that  $y(x)$  satisfies
- $$1 + y'^2 = \frac{B}{y}, \quad \text{where } B \text{ is a constant.} \quad (8 \text{ marks})$$
- ii) Show that  $x$  and  $y$  may be written as
- $$x = a(\theta - \sin\theta), \quad y = a(1 - \cos\theta).$$
- (6 marks)

**QUESTION FIVE (20 Marks)**

- a. A clock starts on the ground and then moves up a tower at constant speed  $v$ . It sits on top of the tower for a time  $T$  and then descends at constant speed  $v$ . If the tower has height  $h$ , how long should the clock sit at the top so that it comes back showing the same time as a clock that remained on the ground? (10 marks)
- b. A spaceship travels at speed  $v$  to a distant star. Upon reaching the star, it decelerates and then accelerates back up to speed  $v$  in the opposite direction (uniformly, and in a short time compared with the total journey time). By what fraction does the traveler age less than her twin on the earth? (Ignore the gravity from the earth.)
- Work in:
- (a) The earth frame.
- (b) The spaceship frame (10 marks)

