JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION

2^{ND} YEAR 1^{ST} SEMESTER 2018/2019

MAIN REGULAR

COURSE CODE: SPH 203

COURSE TITLE: MATHEMATICAL METHODS FOR PHYSICS I

EXAM VENUE: **STREAM: (B.Ed Sc)**

DATE: **EXAM SESSION:**

TIME: 2:00 HRS

INSTRUCTIONS:

1. **Attempt question 1 (compulsory) and ANY other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question 1

(a) **(i)** Compute the limit

$$\lim_{x \to 1} \frac{\sqrt{x} - 1}{\sqrt[3]{x} - 1}$$

(ii) Show that $\cosh^2 x - \sinh^2 x = 1$

(6 Marks)

(b) Find, from first principles, the derivative of the function

$$y = x^{-1}$$

(5 Marks)

(c) Evaluate the following integral,

$$\int x \sin 5x \, \mathrm{d}x$$

(5 Marks)

(d) **(i)** List down any two axioms of a vector space.
(ii) Investigate whether the following is a basis for $\mathbb{R}^2$,

$$\left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \end{pmatrix} \right\rangle$$

(6 Marks)

(e) Determine a scalar c such that the following vectors are orthogonal,

$$\vec{a} = 2\hat{\mathbf{i}} - c\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \qquad \vec{b} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

(3 Marks)

(f) The potential energy between two atoms in a diatomic molecule is given by $U(x) = 2/x^{12} - 1/x^6$. Find the minimum potential energy between the two atoms.

(5 Marks)

Question 2

(a) A curve C has the parametric equations $x = 2t - 5$, $y = t^2 - 4t + 3$. Find an equation of the tangent line to C that is parallel to the line $y = 3x + 1$.

(6 Marks)

(b) A boy wishes to cut a 2.5 m long piece of wire into two pieces. One piece will be bent into a circular shape and the other into the shape of a square. Find the length that each wire should have so that the sum of the areas is a maximum (you may leave your answers in terms of π).

(10 Marks)

(c) Differentiate with respect to x:

(i) $y = 5x^4 - \frac{1}{\sqrt[3]{x}}$

(ii) $y = 2\cos^3 x + \sin 4x$

(4 Marks)

Question 3

(a) Consider the set V of all ordered triples of real numbers (x, y, z). Let the operations $(+)$ and $(\cdot)$ be defined by

$$\begin{aligned} (x, y, z) + (x', y', z') &= (x + x', y + y', z + z') \\ c \cdot (x, y, z) &= (cx, y, z) \end{aligned}$$

Show that V is not a vector space.

(10 Marks)

(b) Use an appropriate method to evaluate,

$$\int \frac{x^2 + 2x + 4}{(x+1)^3}\, \mathrm{d}x$$

(10 Marks)

Question 4

(a) Use de L'Hôpital's rule to evaluate,

$$\lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2}$$

(4 Marks)

(b) A series is given by

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Find the radius and interval of convergence for the series.

(5 Marks)

(c) Given the sine series,

$$\sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

use calculus to obtain the cosine series. Hence or otherwise calculate the cosine of 0.1 radians.

(5 Marks)

(d) The second term of an arithmetic series is 6 and the sixth term is 18. Find the sum of the first ten terms of the series.

(6 Marks)

Question 5

(a) **(i)** State two practical applications of calculus in physics.
(ii) In the adiabatic expansion of air, the pressure P and volume V are related by $PV^{1.4} = k$, where k is a constant. At a certain instant, the pressure is 10000 $\mathrm{N/m^2}$ and the volume is 32 $\mathrm{m^3}$. At what rate is the pressure changing at that instant if the volume is decreasing at a rate of 2 $\mathrm{m^3/s}$?

(8 Marks)

(b) **(i)** Given that $\vec{a} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and $\vec{b} = 3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, evaluate the vector product $\vec{a} \times \vec{b}$.
(ii) Obtain the component of $\vec{a}$ in the direction of $\vec{b}$.

(5 Marks)

(c) A transformation $\vec{L} : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by

$$\vec{L}\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+1 \\ 2y \\ z \end{bmatrix}$$

Determine whether $\vec{L}$ is a linear transformation.

(7 Marks)