JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE

1^{ST} YEAR 1^{ST} SEMESTER 2018/2019

MAIN REGULAR

COURSE CODE: SPH 801

COURSE TITLE: CLASSICAL MECHANICS

EXAM VENUE: **STREAM: (M.Sc)**

DATE: **EXAM SESSION:**

TIME: 3:00 HRS

INSTRUCTIONS:

1. **Attempt question 1 (compulsory) and ANY other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Useful constants

gravitational acceleration, $g, = 9.8$ m s^{-2}

Question 1

(a) A particle is moving radially round a sphere. Using $\hat{e}_r$, $\hat{e}_\theta$, $\hat{e}_\phi$ as the unit vectors in the increasing directions of r, θ, and ϕ respectively in spherical coordinates, derive the expression for velocity and acceleration of the particle.

(5 Marks)

(b) If the Hamiltonian H is independent of t explicitly, show that it is a constant of motion.

(4 Marks)

(c) Consider any two continuous functions of the generalized coordinates and momenta $g(q_k, p_k)$ and $f(q_k, p_k)$.

(i) Define the Poisson brackets for the functions g and f.

(3 Marks)

(ii) Show that

$$\frac{\mathrm{d}g}{\mathrm{d}t} = [g, H] + \frac{\partial g}{\partial t}$$

where H is the Hamiltonian.

(2 Marks)

(d) **(i)** State Hamilton's principle.

(1 Marks)

(ii) Derive Lagrange's equations from Hamilton's principle.

(4 Marks)

(e) A block of mass m slides down a frictionless inclined plane whose lower end is inclined at an angle θ from the horizontal. Taking the distance covered along the plane as x, use the Hamilton's principle to determine the equation of motion of the block.

(5 Marks)

(f) Find the curve $z = z(y)$ which minimizes the function

$$J = \int (z'^2 - 3)\mathrm{d}y$$

where $z(0) = 3$ and $z(2) = 5$

(4 Marks)

(g) Show the physical significance of any two Poisson brackets.

(2 Marks)

Question 2

(a) A projectile is launched in a resistive medium where the retarding force is proportional to the instantaneous velocity of the projectile. The position of the projectile in the xy plane is given as

$$\begin{aligned} x &= \frac{u}{k}\left(1 - e^{-kt}\right) \\ y &= \frac{-gt}{k} + \frac{kv+g}{k^2}\left(1 - e^{-kt}\right) \end{aligned}$$

where k is a constant, u and v are the initial launching velocity components in the x- and y-directions respectively, t is the time and g is the acceleration due to gravity. Use the series expansion method to show that the time of flight T approximates to

$$T \approx \frac{2v}{g}\left(1 - \frac{kv}{3g}\right)$$

(8 Marks)

(b) Derive Lagrange's equations from D'Alembert's principle.

(6 Marks)

(c) A bead slides without friction on a frictionles wire in the shape of a cycloid (see Fig. 1) with equations

$$x = a(\theta - \sin\theta), \quad y = a(1 + \cos\theta) \qquad (0 \le \theta \le 2\pi)$$

Find:

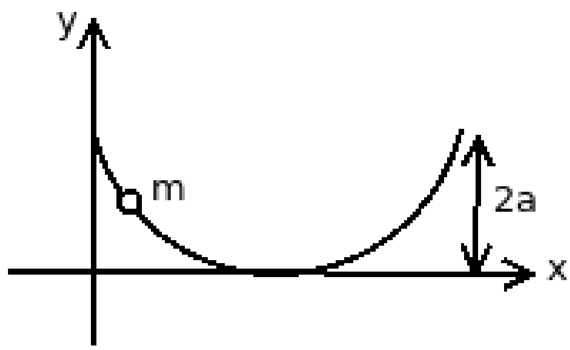

Figure 1:

(i) The Lagrangian of the system.
(ii) The equation of motion of the bead.

(3,3 Marks)

Question 3

(a) Derive Hamilton's canonical equations from Hamilton's principle.

(5 Marks)

(b) A block of mass m slides down a frictionless inclined plane whose lower end is inclined at an angle α from the horizontal. Taking the distance covered along the plane as x, use Hamilton's principle to determine the equation of motion of the block.

(4 Marks)

(c) Given that the function $f = f(z, z', y)$ where $z' = \frac{\mathrm{d}z}{\mathrm{d}y}$, z and y are the dependent and independent variables respectively, derive the Euler-Lagrange equation.

(7 Marks)

(d) A perfectly flexible cable of length L is hanging between points (a, b) and $(-a, b)$ in a vertical xy plane.

(i) What physical quantity would you maximize when getting the equation of the curve of the cable that is an extremum?

(ii) Derive an expression that you would identify with the function f in the Euler-Lagrange equation.

(1,3 Marks)

Question 4

(a) (i) Write down the integral forms of the modified Hamilton's principle in the old and new canonical variables.

(2 Marks)

(ii) Let F be a generating function dependent only on q_k, p_k, t i.e. a second type generating function. Derive the transformation relations for this generating function.

(5 Marks)

(b) (i) Use Poisson brackets to show that the transformation

$$q = \sqrt{\frac{2P}{\sqrt{km}}} \sin Q, \quad p = \sqrt{2P\sqrt{km}} \cos Q$$

is canonical.

(3 Marks)

(ii) Prove that the generating function giving rise to the canonical transformation in (b)(i) is

$$F = \frac{1}{2}\sqrt{km}q^2 \cot Q$$

(4 Marks)

(c) Use the transformation given in (b)(i) to show that the Hamiltonian of a simple harmonic oscillator is cyclic in Q and hence obtain the solution of the harmonic oscillator in terms of q.

(6 Marks)

Question 5

(a) A body of mass m is falling freely in a place with uniform gravitational field. Use the Hamilton-Jacobi method to solve for the motion of the body.

(9 Marks)

(b) Show that if the transformation equations do not contain the time explicitly, then the kinetic energy T is always a homogeneous quadratic form in generalized coordinates.

(4 Marks)

(c) Two particles move in one dimension at the junction of three springs, with spring constants K, $3K$ and K respectively, as shown in Figure 2. The springs have unstretched lengths equal to a, while the two masses are equal.

(i) Set up the secular equation for the system.

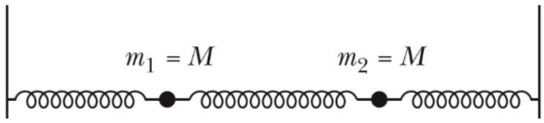

Figure 2:

(4 Marks)

(ii) Find the eigenfrequencies of the system for small vibrations.

(3 Marks)