JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITYDRAFT EXAMINATION FOR BSc IN MATHEMATICS

2nd YEAR 1st SEMESTER 2017/2018 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: IIT 3218

COURSE TITLE :INTRODUCTION TO NUMBER THEORY

EXAM VENUE: **STREAM: BSc Y2S1**

TIME: 2 HOURS **EXAM SESSION:**

Instructions:

Answer question1 and any other two questions

1. **Show all the necessary working**
2. **Candidates are advised not to write on the question paper**

Candidates must hand in their answer booklets to the invigilator while in the examination room

QUESTION 1 (30 MARKS)

(a) State four properties of real numbers. (4 marks)

(b) Suppose a, b and c are integers, prove that

(i) If a/b and b/c, then a/c. (3 marks)

(ii) If a/b, then for any integer x, a/bx (3 marks)

(c) Given that $a = 573$ *and* $b = -16$, find the integers q *and* r such that $a = bq + r$ *and* $0 < r < |b|$. (4 marks)

(d) Let $a = 8316$ and $b = 19800$. Express each number in its prime factors and hence find:

(i) gcd(a,b) (2 marks)

(ii) lcm(a,b) (2 marks)

(e) Construct the addition and multiplication tables for $\mathbb{Z}_5$ (4 marks)

(f) Prove that if gcd(a,b)=1 and that a and b both divide c, then ab divides c. (4 marks)

(g) Solve the congruence equation $8x \equiv 12 \pmod{28}$ (4 marks)

QUESTION 2 (20 MARKS)

(a) Suppose a/b and a/c. Prove that for any integers m and n, $a/(bm + cn)$.(6 marks)

(b) Let $a = -433$ *and* $b = -17$. Find the integers q *and* r such that $a = bq + r$ *and* $0 < r < |b|$. (4 marks)

(c) Determine all the solutions of the integers in the linear Diophantine equation $247x + 91y = 52$. (10 marks)

QUESTION 3 (20 MARKS)

(a) Let m be a positive integer. Prove that:

(i) For any integer a, $a \equiv a \pmod{m}$ (3 marks)

(ii) if $a \equiv b \left(\text{mod}\, m\right)$, then $b \equiv a \left(\text{mod}\ m\right)$. (3 marks)

(b) Let $a = 252 \ \ and \ \ b = 198$.

(i) Using division algorithm find $d =$gcd(a,b), the greatest common divisor of a and b

(6 marks)

(ii) Using Euclidean algorithm, find the integers $x \ \ and \ \ y$ such that $d = xa + yb$

(6 marks)

(iii) Find lcm(a,b), the least common multiple of a and b. (2 marks)

QUESTION 4 (20 MARKS)

(a) Let a and b be ***relatively prime*** integers. Explain what is meant by relatively prime. (2 marks)

(b) Suppose $a \equiv c \ (\text{mod}\, m) \ \ and \ \ b \equiv d \ \left(\text{mod}\, m\right)$. Prove that :

(i) $a + b \equiv c + d \ (\text{mod}\, m)$ (4 marks)

(ii) $a.b \equiv c.d \ (\text{mod}\, m)$ (4 marks)

(c) State the Chinese remainder theorem. (2 marks)

(d) Solve the following systems of linear congruence equation. (8 marks)

$$x \equiv 2 \, (\text{mod}\, 3)$$
$$x \equiv 3 (\text{mod}\, 5)$$
$$x \equiv 4 \, (\text{mod}\, 11)$$

QUESTION 5 (20 MARKS)

(a) Let a and b be two real numbers such that $|a+b| \le |a| + |b|$.

Show that $|a-b| \ge \big||a| - |b|\big|$ (7 marks)

(b) Solve the following congruence equation.

(i) $x^2 - 1 \equiv 0 \left(\text{mod}\, 8\right)$ (4 marks)

(ii) $33x \equiv 38 \ (\text{mod}\, 280)$ (9 marks)