



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE UNIVERSITY EXAMINATION
FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL
1ST YEAR 1ST SEMESTER 2018/2019 ACADEMIC YEAR
MAIN REGULAR

COURSE CODE: SMA 100

COURSE TITLE: BASIC MATHEMATICS

EXAM VENUE:

STREAM: (BSc. Actuarial)

DATE:

EXAM SESSION:

TIME: 2.00 HOURS

Instructions:

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 marks)

- a) Distinguish between the following two sets:
- (i) \emptyset
 - (ii) $\{\emptyset\}$ (4 marks)
- b) List the members of the set:
 $\{x \mid x \text{ is a real number such that } 4x^3 + 5x^2 - 6x = 0\}$ (4 marks)
- c) Find the sum of the following arithmetical progression:
 $7 + 1 - 5 - 11 - \dots - 299$ (3 marks)
- d) Solve the following equation on the interval $0 \leq \theta \leq 2\pi$:
 $3(1 - \cos \theta) = \sin^2 \theta$ (5 marks)
- e) Solve the following equation:
 $\log_{\frac{1}{3}}(x^2 + x) - \log_{\frac{1}{3}}(x^2 - x) = -1$ (6 marks)
- f) Find the coefficient of the term in x^2 in the expansion of:
 $\left(\sqrt{x} + \frac{3}{\sqrt{x}}\right)^8$ (4 marks)
- g) Find $\frac{z}{w}$, leave your answer in polar form:
 $z = 2 + 2i$
 $w = \sqrt{3} - i$ (4 marks)

QUESTION TWO (20 marks)

- a) A piece of string 10 m long is to be cut into pieces, so that the lengths of the pieces form an arithmetic sequence.
- (i) The lengths of the longest and shortest pieces are 1 m and 25 cm respectively; how many pieces are there? (5 marks)
 - (ii) If the same string had been cut into 20 pieces with lengths that formed an arithmetic sequence, and if the length of the second longest had been 92.5 cm, how long would the shortest piece have been? (5 marks)
- b) The fourth, seventh and sixteenth terms of an arithmetical progression are in geometrical progression. If the first six terms of the arithmetical progression have a sum of 12, find the common difference and the common ratio. (10 marks)

QUESTION THREE (20 marks)

- a) Find the power set of $K = \{0, 2, 4, 6\}$ (4 marks)
- b) Let P be the set of students in Education Science class in JOOUST and let Q be the set of students who take Basic Mathematics in first year. Describe the students in each of these sets.
- (i) $P \cup Q$ (1 mark)
 - (ii) $P \cap Q$ (1 mark)
 - (iii) $P - Q$ (1 mark)
 - (iv) $Q - P$ (1 mark)

c) Prove the following distributive law of set operations:

$$F \cup (G \cap H) = (F \cup G) \cap (F \cup H) \quad (4 \text{ marks})$$

d) On a standard three-circle Venn diagram shade the regions corresponding to the given set expressions

(i) $(C \cup A^c) \cap B^c$ (4 marks)

(ii) $(A^c \cap B) \cup (A \cap C)$ (4 marks)

QUESTION FOUR (20 marks)

a) Solve the following equation

$$\sec \theta = \tan \theta + \cot \theta$$

for θ where $0^\circ \leq \theta \leq 360^\circ$ (6 marks)

b) Prove the following identity:

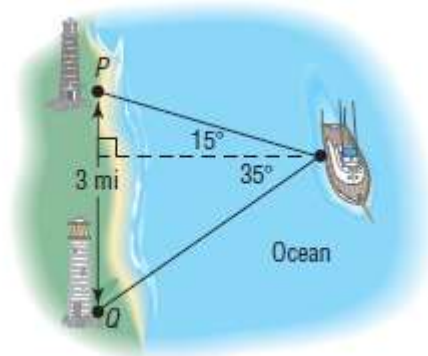
$$\frac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)} = \frac{1 - \tan \alpha \tan \beta}{1 + \tan \alpha \tan \beta} \quad (5 \text{ marks})$$

c) The navigator of a ship at sea spots two lighthouses that she knows to be 3 miles apart along a straight seashore. She determines that the angles formed between two line-of-sight observations of the lighthouses and the line from the ship directly to shore are 15° and 35° . See the illustration.

(i) How far is the ship from lighthouse P? (3 marks)

(ii) How far is the ship from lighthouse Q? (3 marks)

(iii) How far is the ship from shore? (3 marks)



QUESTION FIVE (20 marks)

a) Solve the system of equations

$$2x + y - z = 3$$

$$-x + 2y + 4z = -3$$

$$x - 2y - 3z = 4$$

using Cramer's Rule if it is applicable. If Cramer's Rule is not applicable, say so. (10 marks)

b) The expression $ax^2 + bx + c$ is divisible by $x - 1$, has remainder 2 when divided by $x + 1$, and has remainder 8 when divided by $x - 2$. Find the values of a, b, c . (6 marks)

c) In how many ways can a committee consisting of 2 faculty members and 3 students be formed if 6 faculty members and 10 students are eligible to serve on the committee? (4 marks)