JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

1ST YEAR 1ST SEMESTER 2018/2019 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SMA 3114

COURSE TITLE: ANALYTICAL METHODS FOR COMPUTING

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 marks)

a) Define the following terms:

   i) An injective function

   ii) A proper subset S of a set A.

   iii) Algorithm as used in computing. (6mks)

b) Given that $A = \{x, y, z\}$ and $B = \{a, b\}$ determine

   i) The cardinality of A (1mk)

   ii) The power set of A (3mks)

   iii) The Cartesian product of $B \times A$ (2mks)

c) Given that $f(x) = x^2 + 3$ and $g(x) = \frac{1}{3}x + 1$. Show that $f \circ g \neq g \circ f$. (4mks)

d) Write in tabular form the following set
   $Q = \{x: 2x^2 - 3x - 20 = 0\}$. (4mks)

e) Let $P = 2 + 2i$ and $R = -2 + 4i$. Determine

   i) $\bar{P}$ (1mk)

   ii) $\bar{P}.R$ (3mks)

f) Find the value of the integral $\int_{-2}^{3}(10 + 2x - 2x^3)dx$. (3mks)

g) Find the value of q in the equation $\log_5 q = -3$. (3mks)

QUESTION TWO (20 marks)

a) Let $U = \{1,2,3,4,5,6,7,8,9,10\}, A = \{2,4,6,8,9\}$ and $B = \{1,3,4,7,9\}$. Find

   i) B^c (1mk)

   ii) $A^c \cup B^c$ (2mks)

   ii) $B - A$ (1mk)

b) Given that X and Y are two non-empty sets, prove that
   $(X \cup Y)^c = X^c \cap Y^c$. (4mks)

c) A research conducted on the eating habits among 720 people in Kisumu County, it was found out that:
375 people eat fish,
325 people eat beef,
370 people eat chicken,
205 people eat both fish and beef,
160 people eat both chicken and beef,
155 people eat both chicken and fish,
105 people eat all the three types of food.

i. Present the above information on a Venn diagram. (5mks)
ii. Find the total number of people who eat two types of food only. (2mks)
iii. Find the total number of people who eat one type of food only. (2mks)
iv. Find the total number of people who do not eat any of three types of food. (3mks)

QUESTION THREE (20 marks)

a) Solve the system of linear equations below using Cramer's Rule.

$$\begin{aligned} 2x + y + z &= 1 \\ 3x + z &= 4 \\ x - y - z &= 2 \end{aligned}$$ (8mks)

b) Use Gauss Jordan-row elimination method to solve the following system of linear equations.

$$\begin{aligned} 5x + 2y &= -5 \\ 3x - y &= -14. \end{aligned}$$ (4mks)

c) If $M = \begin{bmatrix} 1 & 0 & 4 \\ 2 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$, determine

i) The adjoint of T. (5mks)

ii) The determinant of T. (2mks)

iii) The inverse of T. (1mks)

QUESTION FOUR (20 marks)

a) Determine the derivative of the following function
$y = e^{-3(x^2+3)}$. (3mks)

b) Determine the area bounded by the curve $y = x^2 - 5x + 4$ and the x −axis. (6mks)

c) The concentration C in mg of a chemical in the bloodstream, t hours after injection into the muscle tissue can be modeled by $C = \frac{3t}{27+t^3}$; $t \geq 0$. Determine the time when the concentration reaches the highest level. (6mks)

d) Solve the triangle ABC given that $AB = 10cm, BC = 7cm$ and $AC = 5cm$. (5mks)

QUESTION FIVE (20 marks)

a) Write the complex number $2 - 2i$ in polar form. Hence use De-Moivre's theorem to evaluate $(2 - 2i)^5$, leaving your answer in the form $a + ib$; $a, b \epsilon \mathbb{R}$. (7mks)

b) Define linearly independent vectors. (2mks)

c) Given that $\boldsymbol{u} = (1,3,-2)$ and $\boldsymbol{v} = (-2,2,-1)$. Determine

i) $2\boldsymbol{u} - \frac{1}{2}\boldsymbol{v}$. (3mks)

ii) $|\boldsymbol{u}|$. (2mks)

iii) $\boldsymbol{v} \cdot \boldsymbol{u}$ (3mks)

d) Given that the vectors $\boldsymbol{u} = (-4, k)$ and $v = (-2,3)$ are perpendicular. Find k. (3mks)