JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION ARTS, SPECIAL EDUCATION AND EDUCATION SCIENCE

2ND YEAR 1ST SEMESTER 2018/2019 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SMA 210

COURSE TITLE: PROBABILITY AND DISTRIBUTION THEORY I

EXAM VENUE: **STREAM: (B.e.d ARTS, SPECIAL ed. & B.ed SCIENCE)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

<u>Instructions:</u>

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) Let X and Y have a bivariate probability density function given by

$$f(x,y)=\begin{cases} \frac{3}{2}x^2 & 0 \le x \le 2; 0 \le y \le 1 \\ 0 & otherwise \end{cases}$$

Obtain marginal densities of X and Y. (4 Marks)

b) Suppose that the joint probability distribution function of X and Y is

$$f(x,y)=\begin{cases} \frac{3}{16}(4-2x-y) & x>0; y>0; 2x+y<4 \\ 0 & otherwise \end{cases}$$

Determine:

i. The conditional probability density function of Y given X. (4 Marks)

ii. Compute $\Pr[Y \ge 2 / X = 0.5]$ (4 Marks)

c) Outline TWO properties of covariance of two random variables. (2 Marks)

d) Suppose that X and Y are random variables of $\mathrm{var}(X)=9$, $\mathrm{var}(Y)=4$ and $\rho_{XY}=-\frac{1}{6}$.

Determine:

i. $\mathrm{var}(X+Y)$ (2 Marks)

ii. $\mathrm{var}(X-3Y+4)$ (2 Marks)

e) Given that X_1 and X_2 are random variables with joint probability distribution function given by

$$f(x_1,x_2)=\begin{cases} x_1+x_2 & 0<x_1<1, 0<x_2<1 \\ 0 & otherwise \end{cases}$$

Determine whether or not X_1 and X_2 are independent. (5 Marks)

f) Consider a two dimensional random variable (X_1, X_2) having a density function given by

$$f(x_1,x_2)=\begin{cases} 8x_1x_2 & 0 \le x_1 \le 1; 0 \le x_2 \le 1 \\ 0 & otherwise \end{cases}$$

Compute:

i. $E(3X_1+2X_2)$ (4 Marks)

ii. $E(X_1X_2)$ (3 marks)

QUESTION TWO (20 MARKS)

a) Suppose that X is a random variable such that $0<\delta_X^2<\infty$ and that $Y=aX+b$ for some constant a and b where $a \ne 0$. Show that if $a>0$ then $\rho_{XY}=1$ and if $a<0$ then $\rho_{XY}=-1$ (4 Marks)

b) Describe the regression between X and Y from a joint probability distribution function given by

$$f(x,y)=\begin{cases}\frac{1}{2}xy & 0<y<x:0<x<2\\ 0 & otherwise\end{cases}$$ (16 Marks)

QUESTION THREE (20 MARKS)

a) Show that the moment generating function of a bivariate normal distribution is given by
$m(t_1,t_2)=\exp\left\{t_1\mu_x+t_2\mu_y+\frac{1}{2}\left[t_1^2\delta_x^2+2\rho t_1t_2\delta_x\delta_y+t_2^2\delta_y^2\right]\right\}$ (10 Marks)

b) Show that if X and Y are random variables with a bivariate normal distribution, then $E(X)=\mu_x$, $E(Y)=\mu_y$, $\text{var}(X)=\delta_x^2$, $\text{var}(Y)=\delta_y^2$ and $\text{cov}(XY)=\rho\delta_x\delta_y$ (10 Marks)

QUESTION FOUR (20 MARKS)

a) Consider two independent random variables X_1 and X_2 both coming from a population with probability density function

$$f(x)=\begin{cases}1 & 0<x<1\\ 0 & otherwise\end{cases}$$

Suppose we define two other random variables $Y_1=X_1+X_2$ and $Y_2=X_1-X_2$. Obtain;

i. the joint probability distribution of Y_1 and Y_2

ii. the marginal probability distribution function Y_1 (10 Marks)

b) Define a Beta distribution. (2 Marks)

c) Obtain the mean and variance of a Beta distribution. (8 Marks)

QUESTION FIVE (20 MARKS)

Suppose that X_1 and X_2 are jointly distributed random variables with probability distribution function given by

$$f(x_1,x_2)=\begin{cases}\frac{1}{8}(x_1+x_2) & 0\le x_1\le 2;0\le x_2\le 2\\ 0 & otherwise\end{cases}$$

Compute the coefficient of correlation between X_1 and X_2