JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE

2018/2019 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SAS 305

COURSE TITLE: STOCHASTIC PROCESSES 1

EXAM VENUE: **STREAM: ACTUARIAL SCIENCE**

DATE: **EXAM SESSION: DECEMBER 2018**

TIME: 2.00 HOURS

Instructions:

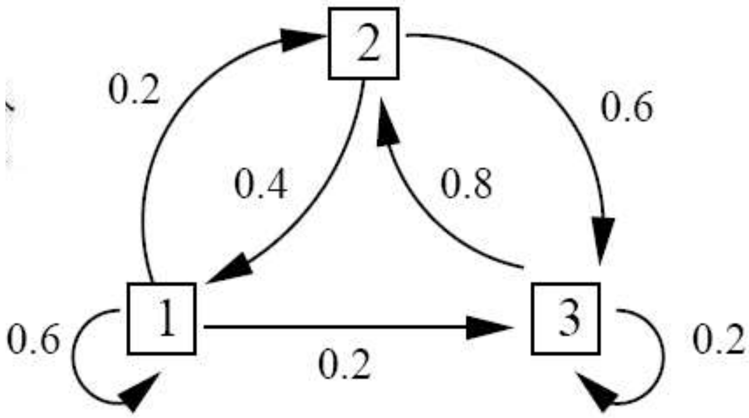
1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question One Compulsory (30mks)

a) Briefly define the following terms as used in stochastic processes (12mks)
   i) Stochastic Process
   ii) State Space
   iii) Time space
   iv) Bernoulli process
   v) Poisson process
   vi) Transition matrix

b) My newspaper is delivered late 60 percent of the time. Suppose deliveries can be described as a Bernoulli process and define a random variable X to be the number of late deliveries per year (365 days).
   i) Find the mean of X
   ii) Find the standard deviation of X. (4mks)

c) Consider the following transition diagram below.



i) Find the transition matrix, P. (2mks)

ii) Find $P(X_2 = 3 | X_0 = 1)$. (3mks)

iii) Suppose that Purpose-flea is equally likely to start on any vertex at time 0. Find the probability distribution of X_1. (3mks)

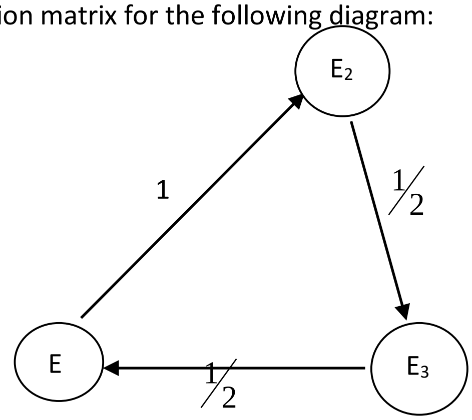
iv) Suppose that Purpose-flea begins at vertex 1 at time 0. Find the probability distribution of X_2. (3mks)

v) Suppose that Purpose-flea is equally likely to start on any vertex at time 0. Find the probability of obtaining the trajectory (3, 2, 1, 1, 3). (3mks)

Question Two (20mks)

i) An insurance policy on an electrical device pays a benefit of 4000 if the devices fails during the first years. The amount of the benefit decreases by 1000 each successive year until it reached 0. If the device has not failed by the beginning of any given year, the probability of failure during that year is 0.4. what is the expected benefit under this policy (5mks)

ii) i. Give a transition matrix for the following diagram:



ii. Draw a transition graph (diagram) for the following transition matrix. (5mks)

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

iii) In modeling the number of claims filed by an individual under an automobile policy during a three-year period, an actuary makes the simplifying assumption that for all integers $n \geq 0$, $P_n+1 = (1/5)P_n$, where P_n represents the probability that the policyholder files n claims during the period.

Under this assumption, what is the probability that a policyholder files more than one claim during the period? (5mks)

iv) A company prices its hurricane insurance using the following assumptions: i. In any calendar year, there can be at most one hurricane. ii. In any calendar year, the probability of a hurricane is 0.05. iii. The number of hurricanes in any calendar year is independent of the number of hurricanes in any other calendar year. Under the company's assumptions, find the probability that in a 20-year period a) there is at least one hurricane (5mks)

Question Three (20mks)

Customers arrive in a bank according to a Poisson process with rate ¸λ = 5 per hour. Given that the store opens at 9:00am,

i. what is the probability that exactly one customer has arrived by 9:30? (4mks)
ii. what is the probability that five have arrived by 11:30? (4mks)
iii. given 1, what is the probability that total of five have arrived by 11:30? (6mks)
iv. given 1 and 2, what is the probability that the total of 10 has arrived by the time the store closes (5:00pm)? (6mks)

Question Four (20mks)

A company takes out an insurance policy to cover accidents that occur at its manufacturing plant. The probability that one or more accidents will occur during any given month is 3/5. The number of accidents that occur in any given month is independent of the number of accidents that occur in all other months.

Compute the Probability Mass Function (5mks)

Calculate the probability that there will be at least four consecutive months in which no accidents occur. (15mks)

Question Five (20mks)

a) The leading brewery on the West Coast (A) has hired a TM specialist to analyze its market position. It is particularly concerned about its major competitor (B). The analyst believes that brand switching can be modeled as a Markov chain using 3 states, with states A and B representing customers drinking beer produced from the aforementioned breweries and state C representing all other brands. Data are taken monthly, and the analyst has constructed the following one-step transition probability matrix.

A	B	C
0.7	0.2	0.1
0.7	0.75	0.05
0.1	0.1	0.8

What are the steady-state market shares for the two major breweries? (10mks)

b) You are given the following information:

• Mortality for an individual can be described using a non-homogenous Markov Chain process with two states:

State 1: Alive

State 2: Deceased

You are given the following transition probability matrices for this individual:

$$Q_0 = \begin{pmatrix} 0.9 & 0.1 \\ 0 & 1 \end{pmatrix} \qquad Q_1 = \begin{pmatrix} 0.8 & 0.2 \\ 0 & 1 \end{pmatrix} \qquad Q_2 = \begin{pmatrix} 0.7 & 0.3 \\ 0 & 1 \end{pmatrix} \qquad Q_3 = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$$

- An insurance policy is issued to this individual at time 0.

- The insured is in state 1 at the time the policy is issued.
- A benefit of $100,000 is paid out upon transition of the insured from state 1 to state 2.
- Transitions occur at the end of each time period.
- The insurance company receives a premium of $25,000 at the beginning of each time period, if the insured is in state 1 at that time.
- $i = 5\%$

Calculate the benefit reserve for this policy at time 1, assuming the insured is in state 1 and the premium for this time period has not yet been paid. (10mks)