JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE UNIVERSITY EXAMINATION

FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

3ST YEAR 1ST SEMESTER 2018/2019 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SMA 301 **COURSE TITLE: ODE**

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY) (30 marks)

a) Determine:

i) the order,

ii) the degree,

iii) the unknown function, and

iv) the independent variable

for differential equation

$$\left(\frac{d^4y}{dx^4}\right)^2 + x^3\left(\frac{d^3y}{dy^3}\right)^5 = \sin x \quad \text{(4 marks)}$$

b) Find the values of c_1 and c_2 so that the given function satisfies the given prescribed initial conditions:

$y(x) = \mathrm{c_1x + c_2 + x^2 - 1}$; $y(1) = 1$, $y'(1) = 2$ (6 marks)

c) Show that the solution of the equation $y'' + y = 0$ is $y = c_1 \sin x + c_2 \cos x$, where c_1 and c_2 are arbitrary constants(4 marks)

d) Form the differential equation representing the family curves

$y = a\cos(\mathrm{x+b})$ (4 marks)

e) Solve $(x+2)\dfrac{dy}{dx} = x^2 + 4x - 5$. (4 marks)

f) Show that a separable first order differential is always exact. (4 marks)

g) A certain city had a population of 200000 in 1970 and a population of 350000 in 1980. Assume that its population will continue to grow exponentially at a constant rate. What populations can its city planners expect in the year 2020? (4 marks)

QUESTION TWO (20 marks)

a) Solve the following differential equation

$$\cos^2 x\frac{dy}{dx} + y = \tan x \quad \text{(6marks)}$$

b) Solve the initial value problem:

$$y'' + 2y' = te^{-t} \ ; \ y(0) = 6, \ y'(0) = -1.$$

State the largest interval in which the solution is guaranteed to uniquely exist. (7marks)

c) Solve the initial value problem

$$y'' - y' - 2y = 0, \ y(0) = 2, \ y'(0) = 7. \quad \text{(7 marks)}$$

QUESTION THREE (20 marks)

a) Determine whether or not $\frac{y}{x}dx+\left(y^3+\ln x\right)dy=0$ is exact. If exact, find the solution. (7marks)

b) Find the solution of the given differential equation:

$y'+2y=y^2e^x$; $y\left(0\right)=2$ (6marks)

c) Show that

$\left(x^2+2xy-y^2\right)dx+\left(y^2+2xy-x^2\right)dy=0$; $y\left(1\right)=-1$

is homogeneous and find its solution. (7 marks)

QUESTION FOUR (20 marks)

a) Solve the initial-value problem using the method of undetermined coefficients

$y''-3y'-4y=-8e^t\cos 2t,\quad y\left(0\right)=1,\quad y'\left(0\right)=2.$ (12 marks)

b) Solve the differential equation using the method of variation of parameters

$4y''-4y'+y=16e^{\frac{t}{2}}$ (8 marks)

QUESTION FIVE (20 marks)

a) A particle moves vertically under the force of gravity against air resistance Kv^2, where K is a constant. The velocity at any time is given by the differential equation

$$\frac{dv}{dt}=g-Kv^2$$

If the particle starts off from rest, show that

$$v=\frac{\lambda\left(e^{2\lambda kt}-1\right)}{\left(e^{2\lambda kt}+1\right)}$$

Such that $\lambda=\sqrt{\frac{g}{K}}$. Then find the velocity as the time approaches infinity. (10 marks)

b) Equation $y''+9y=14\sin 4t$ describes a spring block system that is driven by an oscillatory external for $f\left(t\right)=14\sin 4t$ in the absence of friction. If the block has an initial position $y\left(0\right)=4$ and an initial velocity $y'\left(0\right)=1$. Find the solution of the initial value problem. (10 marks)