JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

2ND YEAR 1ST SEMESTER 2018/2019 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SAC 208

COURSE TITLE: RISK THEORY **EXAM VENUE:**

STREAM: (BSc. Actuarial)

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION 1 [COMPULSORY] [30 Marks]

(a) Define the moment generating function of a random variable X and state its properties. **[6 Marks]**

(b)The claim experience of 4000 policies, each exposed to risk for a year, is shown in the table below. Compare the numbers of policies yielding 0,1,2,3 claims with those that would result assuming the number of claims per policy was a Poisson variable with parameter $\lambda = 0.2$

Number of claims	Observed No. of Policies
0	3288
1	642
2	66
3	4
Total	4000

[8 Marks]

(c) The table below indicates claims under householder's policies covering fire, theft and public liability.

	No. of Policies	Period of Observation(Years)	No. of Claims
Fire	1250	1	12
Theft	2500	1	250
Liability	2000	4	35

For one year 200 insurance policies are observed and in that period 26 claims are made in respect of them. Estimate the probability that in the next year there will be less than 750 claims arising from the 6000 policies in the insurance company's portfolio. Assume normal approximation to the Poisson distribution. [**6 Marks**]

(d) Define S to be $X_1 + X_2 + ... + X_N$. It is known that N has a Poisson distribution with parameter 2.8 and X has the following p.d.f;

$$f_X(x) = kx^2 \;\; ; \;\; 4 < x < 8$$

where k is a constant.

Calculate the mean and the variance of S. [**10 Marks**]

QUESTION 2 [20 Marks]

(a)A random variable X follows a gamma distribution with parameters α and λ

(i) Derive the moment generating function (MGF) of X. [**8 Marks**]

(ii) Derive the coefficient of skewness of X. [**6Marks**]

(b)An Actuary for an automobile insurance company determines that the distribution of the annual number of claims for an insured chosen at random is modeled by the negative binomial distribution with mean 0.2 and variance 0.4. The number of claims for each individual insured has a Poisson distribution and the means of these Poisson distributions are gamma distributed over the population of insureds.Calculate the variance of this gamma distribution. [**6 Marks**]

QUESTION 3 [20 Marks]

(a) State the general expression of the exponential families of distributions and use this to derive the relevant expressions for the mean and the variance of these distributions. [**8Marks**]

(b) Extend the result in (a) to obtain an expression for the third central moment. [**4 Marks**]

(c) Show that the following density function belongs to the exponential family of distributions:

$$f(x) = \frac{\alpha^\alpha}{\mu^\alpha \Gamma \alpha} x^{\alpha-1} e^{-x\frac{\alpha}{\mu}}$$

[**4 Marks**]

(d) Using the results in (a) and (b) obtain the second and third central moments for this distribution. [**4 Marks**]

QUESTION 4[20 Marks]

The number of claims, N, in a given year on a particular type of insurance policy is given by:

$$P(N = n) = 0.8 \times 0.2^n, \qquad n = 0, 1, 2, ...$$

Individual claim amounts are independent from claim to claim and follow a Pareto distribution with parameters $\alpha = 5$ and $\lambda = 1,000$.

(a) Calculate the mean and variance of the aggregate annual claims per policy.. [**8 Marks**]

(b) Calculate the probability that aggregate annual claims exceed 400 using:

(i) a Normal approximation.

(ii) a Lognormal approximation. [**10 Marks**]

(c)Explain which approximation in part (a) above you believe [**2 Marks**]

QUESTION 5 [20 Marks]

(a) Define the concept of excesses and excess of loss reinsurances as applied in general insurance. [**4 Marks**]

(b) An insurer underwriting risks with the claim size distribution.

$$F(x) = 1 - e^{-0.001x}$$

proposes the introduction of a Kshs 500 excess. Calculate the mean amount payable by the insurer in respect of all losses. [**8 Marks**]

(c) If instead of the arrangements in (b) above, the insurer had made a reinsurance arrangement so that should a claim exceed Kshs. 3,000, the excess is paid by the insurer. Calculate the mean net sum paid by the insurer after allowing for this reinsurance. [**8 Marks**]