JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

3RD YEAR 2ND SEMESTER 2018/2019 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SAC 300

COURSE TITLE: FINANCIAL MATHEMATICS II

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. Answer question 1 (Compulsory) and ANY other 2 questions
2. Candidates are advised not to write on the question paper.
3. Candidates must hand in their answer booklets to the invigilator while in the examination room.

QUESTION 1 [COMPULSORY] [30 Marks]

(a)Define interest rate risk. [**4 Marks**]

(b)(i)The one-year forward rate applying at a particular point in time t is defined as $f_{t,1}$.

If $f_{t,1} = 4\%$, calculate the continous time forward contract rate $F_{t,1}$ applying over the same period. [**3 Marks**]

(ii) Define the instantaneous forward rate F_t [**3 Marks**]

(c)An investor purchases a bond on the issue date at a price of Kshs.96 per Kshs.100 nominal.

Coupons at an annual rate of 4% are paid annually in arrears. The bond will be redeemed at par 20 years after the issue date. Calculate the gross redemption yield from the bond. [**5 Marks**]

(d) An insurance company calculates the single premium for a contract paying Kshs.10,000 in ten years time as the present value of the benefit payable, at the expected rate of interest it will earn on its funds. The annual effective rate of interest over the whole of the next ten years will be 7%, 8% or 10% with probabilities 0.3,0.5 and 0.2 respectively.

(i)Calculate the single premium. [**5 Marks**]

(ii)Calculate the expected profit at the end of the term of the contract.

[**4 Marks**]

(e) An investment bank models the expected performance of its assets over a five day period. Over that period, the return on the bank's portfolio, i, has a mean value of 0.1% and standard deviation 0.2%. $1+i$ is lognormally

distributed. Calculate the value of j such that the probability that i is less than or equal to j is 0.05 **[6 Marks]**

QUESTION 2 [20 Marks]

(a) If $F_{t,r}$ is the force of interest corresponding to the annual forward rate of interest $f_{t,r}$ and P_t is the price of a unit zero-coupon bond of term t. Show that

$$f_{t,r} = e^{F_{t,r}} - 1$$

and that

$$F_{t,r} = \frac{1}{r} log \left(\frac{P_t}{P_{t+r}} \right)$$

[5 Marks]

(b)The n-year forward rate for transaction beginning at time $t + n$ is denoted by $f_{t,n}$. You are given

$$f_{0,1} = 6.0\% p.a$$
$$f_{0,2} = 6.5\% p.a$$
$$f_{1,2} = 6.6\% p.a$$

Determine the 3-year par yield **[5 Marks]**

(c)Two bonds paying annual coupons of 5% in arrear and redeemable at par have terms of maturity of exactly one year and two years respectively. The gross redemption yield from the 1-year bond is 4.5% per annum effective; the gross redemption yield from the two-year bond is 5.3% per annum

effective. You are informed that the 3-year par yield is 5.6% per annum. Calculate all zero-coupon yields and one-year forward rates implied by the yields given above. **[10 Marks]**

QUESTION 3 [20 Marks]

In a particular bond market, n-year spot rates can be approximated by the function

$$0.06 - 0.02e^{-0.1n}$$

(a) Calculate the gross redemption yield for a 3-year bond which pays coupons of 3% annually in arrear and is redemable at par. **[12 Marks]**

(b)Calculate the four year par-yield. **[8 Marks]**

QUESTION 4 [20 Marks]

A pension fund holds an asset with current value Kshs.1,000,000. The investment return on the asset in a given year is independent of returns all over the years. The annual investment return in the next year will be 7% with probability 0.5 and 3% with probability 0.5. In the second and subsequent years, annual investment returns would be 2%, 4% or 6% with probabilities 0.3, 0.4 and 0.3 respectively

(a) Calculate the expected accumulated value of the asset after 10 years.

[**10 Marks**]

(b) Calculate the standard deviation of the accumulated value of the asset after 10 years, showing all steps in your calculations.   [**10 Marks**]

QUESTION 5[20 Marks]

The liabilities of a fund consists of two lump sum payments due at known times in the future. The second lump sum is due for payment 5 years after the first and is twice the amount of the first.

(a) If the total present value and discounted mean term of the liabilities (both calculated using a market interest rate of 6% p.a effective) are Kshs.75,000 and 8 years respectively determine the timing and the amounts of payments.   [**10 Marks**]

(b) If the assets of the fund consist of a single zero-coupon bond that will mature in 8 years from now with redemption payment of Kshs.119,540, what can you say about this portfolio on the basis of Redington's theory of immunization?   [**10 Marks**]