JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

4^{TH} YEAR 2^{ND} SEMESTER 2018/2019 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SAC 406

COURSE TITLE: RISK AND CREDIBILITY THEORY

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION 1 [COMPULSORY] [30 Marks]

(a) Define partial and full credibility. [**6 Marks**]

(b)Each of r independent risks has a probability 0.25 that a claim is made in a year and 0.75 that no claim is made. Claim sizes are independent with mean 500 and variance 125. Determine the expected value and the variance of the total claimed in a year. [**8 Marks**]

(c)The number of claims per month Y arising on a certain portfolio of insurance policies is to be modelled using a modified geometric distribution with probability density given by

$$p(y|\alpha) = \frac{\alpha^{y-1}}{(1+\alpha)^y}, y = 1, 2, 3, \ldots ...$$

where is an unknown positive parameter. The most recent four months have resulted in claim numbers of 8, 6, 10 and 9.

Derive the maximum likelihood estimate of α. [**4 Marks**]

(d)The number of claims per month are independent Poisson random variables with mean λ and the prior distribution for λ is exponential with mean 0.2. Determine the posterior distribution for λ given the observed values $x_1, x_2, ..., x_n$ of the number of claims in n months and hence the Bayesian estimate of λ under the quadratic loss. [**6 Marks**]

(e)A sample of 100 claims on a general insurance in respect to a certain class of business has a mean of 1216 and variance 362,944. The claim frequency rate is about 0.015. Calculate the minimum size of the portfolio if full credibility $(k = 0.05, p = 0.9)$ is to be assigned to the experience.

[**6 Marks**]

QUESTION 2[20 MARKS]

The last ten claims(in shillings) under a particular class of insurance policy were

$$1330, 201, 111, 2368, 617, 309, 35, 4685, 442, 843$$

(a)Assuming that claims came from a lognormal distribution with parameters μ and σ, derive the formular for the maximum likelihood estimate of these parameters and estimate the parameters using the observed data.

[**12 Marks**]

(b)Assuming that the claim come from a Pareto distribution with parameters α and λ, use the method of moments to estimate these parameters.

[**8 Marks**]

QUESTION 3[20 MARKS]

Claims arrive in a Poisson process at rate λ, and $N(t)$ is the number of claims arriving at time t. The claim amounts are independent random variables $X_1, X_2, ...,$ with mean μ, independent of the arrival process.
The initial surplus is U and the premium loading factor is θ.
(a)Give an expression for the surplus process $U(t)$ at time t. [**3 Marks**]

(b)Define the probability of ruin with the initial surplus $U, \Psi(u)$. State the value of $\Psi(u)$ when $\theta = 0$. [**4 Marks**]

(c)Comment on the statement "As the value of λ increases the probability of ruin must also increase". [**3 Marks**]

(d)A particular portfolio of insurance policies gives rise to aggregate claims which follow a Poisson process with parameter $\lambda = 25$. The distribution of individual claim amounts is as follows:

Claim	50	100	200
Probability	30%	50%	20%

The insurer initially has a surplus of 240. Premiums are paid annually in advance. Calculate approximately the smallest premium loading such that the probability of ruin in the first year is less than 10%.

[**10 Marks**]

QUESTION 4[20 MARKS]

The table below shows aggregate annual claim statistics for three risks over a period of seven years. Annual aggregate claims for risk i in year j are denoted by $X_{i,j}$.

Risk i	$\bar{X} = \frac{1}{7}\sum_{j=1}^{7} X_{ij}$	$S_i^2 = \frac{1}{6}\sum_{j=1}^{7}(X_{ij} - \bar{X}_i)^2$
1	127.9	335.1
2	88.9	65.1
3	149.7	33.9

(a) Calculate the credibility premium of each risk under the assumptions under the assumptions of EBCT model 1. [**15 Marks**]

(b)Explain why the credibility factor is high in this case. [**5 Marks**]

QUESTION 5[20 MARKS]

The total claim amount per annum on a particular insurance policy follows a normal distribution with unknown mean θ and variance 200^2. Prior belief about θ are described by a normal distribution with mean 600 and variance 50^2.Claim amounts $x_1, x_2, ..., x_n$ are observed over n years.

(a) State the posterior distribution of θ. [**6 Marks**]

(b) Show that the mean of the posterior distribution θ can be written in the form of a credibility estimate. [**8 Marks**]

(c) Now suppose that $n = 5$ and that the total claim amounts over the five years were 3400.Calculate the posterior probability that θ is greater than 600. [**6 Marks**]