JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION ARTS, SPECIAL EDUCATION AND EDUCATION SCIENCE

3RD YEAR 2ND 2018/2019 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SAS 310

COURSE TITLE: STOCHASTIC DECISION MODEL I

EXAM VENUE: **STREAM: (B.sc ACTUARIAL SCIENCE)**

DATE: 26/4/19 **EXAM SESSION: 3.00 – 5.00PM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) Outline FOUR quantities of interest for queueing models. (4 Marks)

b) Suppose that customers arrive at a poisson rate of one per every 12 minutes and that the service time is exponential at a rate one service per 8 minutes. Calculate the four quantities of interest in a queueing model. (8 Marks)

c) Calculate the preceding equation of interest if;

i. $\lambda = 1, \mu_1 = 1, \mu_2 = 2$ (5 Marks)

ii. $\lambda = 1, \mu_1 = 2, \mu_2 = 1$ (5 Marks)

d) Differentiate between an open system and closed system in a queueing process (2 Marks)

e) Consider a system of two servers where customers from outside the system arrive at the server 1 at a poisson rate 4 and at server 2 at a poisson rate 5. The server rates of 1 and 2 are respectively 8 and 10. A customer upon completion of the service at server 1 is equally likely to go to server 2 or leave the system that is to say $p_{11} = 0$ and $p_{22} = 1/2$, where as a departure from server 2 will go 25% of the time to server 1 and will depart the system, otherwise $p_{21} = 1/4$, $p_{22} = 0$. Determine the limiting probabilities L and W . (6 Marks).

QUESTION TWO (20 MARKS)

a) Suppose that it costs $c\mu$ dollars per hour to provide service at a rate μ . Suppose also that we incur a gross profit of A dollars for each customers served. If the system has a capacity N . What service rate μ maximizes our total profits if $N = 2$, $\lambda = 1$, $A = 10$, $c = 1$? (10 Marks)

b) Suppose that potential customers arrive at a single-server bank in accordance with a poisson process having rate λ . However, suppose that the potential customer will enter the bank only if the server is free when he arrives. That is, if there is already a customer in the bank, then our arriver, rather than entering the bank will go home. If we assume that amount of time spent in the bank by an entering customer is a random variable having a distribution G then

i. What is the rate at which customers enters the bank?

ii. What proportion of potential customers actually enter the bank?

QUESTION THREE (20 MARKS)

Consider a machine that can be in one of the three states, good condition, fair condition or broken condition. Suppose that a machine in good condition will remain this way for a mean time μ_1 and then will go to either the fair condition or broken condition with probabilities $3/4$ and $1/4$. A machine with fair condition will remain that way for a mean time μ_2 and then will breakdown. A broken machine will be repaired, which takes a mean time μ_3, and when repaired will be in good condition with a probability of $\frac{2}{3}$ and a fair condition with probability of $\frac{1}{3}$. What proportion of time is the machine in each state? If $\mu_1 = 5$, $\mu_2 = 2$, $\mu_3 = 1$, give the proportion of the machine at good condition, fair condition and broken condition.

QUESTION FOUR (20 MARKS)

Show that W is smaller in a $M/M/1$ model having arrivals at rate λ and service at rate 2μ than it is in a two-server $M/M/2$ model with arrivals at rate λ and with each server at rate μ. Can you give an intuitive explanation for this results? Would it also be true for W_Q?

QUESTION FIVE (20 MARKS)

a) Show that if $X_1, X_2, \ldots.$ are independent and identically distributed random variables having finite expectations, and if N is a stopping time for $X_1, X_2, \ldots\ldots$ such that $E[N] < \infty$, then

$$E\left[\sum_{1}^{N} X_n\right] = E[N]E[X]$$ (10 Marks)

b) For a non-homogeneous Poisson process with intensity functions $\lambda(t), t \geq 0,$ where $\int_0^\infty \lambda(t)dt = \infty,$ let $X_1, X_2, \ldots.$ denote the sequence of times at which events occur.

i. Show that $\int_0^{X_1} \lambda(t)dt$ is exponential with rate 1.

ii. Show that $\int_{X_{i-1}}^{X_i} \lambda(t)dt, i \geq 1,$ are independent exponentials with rate 1, where $X_0 = 0$

(10 Marks)