JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

2ND YEAR 1ST SEMESTER 2018/2019

REGULAR (MAIN)

COURSE CODE: SMA 210

COURSE TITLE: PROBABILITY AND DISTRIBUTION THEORY I

EXAM VENUE: **STREAM: (Bed. Arts, Bed Science, Bed Arts (Special Needs) Bsc Actuarial Science)**

DATE: 21/08/19 **EXAM SESSION: 3.00 – 5.00pm**

TIME: 2.00 HOURS

<u>Instructions:</u>

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) Suppose X and Y are discrete random variables with joint distribution

$$f(x,y)=\begin{cases}\frac{1}{30}(x+y) & x=0,1,2,3:y=0,1,2\\ 0 & Otherwise\end{cases}$$

i. Verify that $f(x,y)$ is probability distribution. (3 Marks)

ii. Compute $P(X \leq 2)$ (3 Marks)

b) Let X and Y have the bivariate densities given by

$$f(x,y)=\begin{cases}\frac{3}{2}y^2 & 0 \leq x \leq 2; 0 \leq y \leq 1\\ 0 & Otherwise\end{cases}$$

Obtain $f_1(x)$ and $f_2(x)$ (4 Marks)

c) Given that $f(x,y)=\begin{cases}\frac{1}{54}(x+y) & x=1,2,3:y=1,2,3,4\\ 0 & Otherwise\end{cases}$

i. Determine the conditional distribution of Y given that $X=x$ (3 Marks)

ii. Calculate

- $P(Y=1/X=1)$ (2 Marks)
- $P(Y=4/X=3)$ (2 Marks)

d) Given that $f(x,y)=\begin{cases}(x+y) & 0<x<1:0<y<1\\ 0 & Otherwise\end{cases}$

Determine whether or not X and Y are independent. (6 Marks)

e) Suppose that X is a random variable such that $0<\delta_X^2<\infty$ and that $Y=aX+b$ for some constants a and b where $a=0$. Show that if $a>0$ then $\rho_{XY}=1$ and that if $a<0$ then $\rho_{XY}=-1$ (7 Marks)

QUESTION TWO (20 MARKS)

a) If X and Y are random variables such that $\text{var}(X)<\infty$ and , show that

i. $\text{var}(X+Y)=\text{var}(X)+\text{var}(Y)+2\text{cov}(XY)$

ii. $\text{var}(aX+bY)=a^2\text{var}(X)+b^2\text{var}(Y)+2ab\text{cov}(XY)$ (8 Marks)

b) Suppose that X and Y are random variables of $\text{var}(X)=9$; $\text{var}(Y)=4$ and $\rho_{XY}=-\frac{1}{6}$

Determine

i. $\text{var}(X+Y)$

ii. $\text{var}(X-3Y+4)$

iii. $\text{var}(3X-2Y)$ (8 Marks)

c) Explain TWO properties of covariance and correlation (4 Marks)

QUESTION THREE (20 MARKS)

If X and Y have discrete joint distribution for which the joint probability distribution is defined as follows:

$$f(x,y)=\begin{cases} \frac{1}{30}(x+y) & x=0,1,2,:\ y=0,1,2,3 \\ 0 & \text{Otherwise} \end{cases}$$

Determine the marginal probability function of X and Y and represent them in tabular form.

QUESTION FOUR (20 MARKS)

Let X and Y be continuous random variable with joint probability distribution function

$$f(X,Y)=\begin{cases} e^{-Y} & 0<X<Y<\infty \\ 0 & \text{otherwise} \end{cases}$$

Determine;

i) $M(t_1,t_2)$.
ii) $E(X)$, $\text{var}(X)$.
iii) $E(Y)$, $\text{var}(Y)$.
iv) $\text{cov}(X,Y)$.
v) The correlation between X and Y.
vi) The marginal moment generating functions of X and Y.
vii) Are X and Y independent?

QUESTION FIVE (20 MARKS)

a) The joint probability density function of continuous random variables X and Y is

$$f(x,y)=\begin{cases} \frac{1}{8}(6-x-y) & 0<x<2:2<y<4 \\ 0 & \text{Otherwise} \end{cases}$$

Find the joint distribution function of X and Y. Hence or otherwise find

i. $P(X<1,Y<3)$
ii. $P(X<1)$
iii. $P(Y\le 3)$ (10 Marks)

b) The joint probability distribution function of continuous random variables X and Y is given by

$$f(x,y)=\begin{cases} e^{-x} & 0<y<x<\infty \\ 0 & \text{Otherwise} \end{cases}$$

Find the probability distribution function of $U=X+Y$ and $V=X-Y$ (10 Marks)